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Three paradoxes: circles and singularities

(I). Three paradoxes

Suppose I write on the board the following sentence:

(L) The sentence written on the board in room 213 is not true.

I believe that room 213 is the room next door, and that an untrue sentence is written on the board there. But I am confused about my whereabouts, and I am in room 213. Under these circumstances, (L) is paradoxical. For suppose (L) is true; then it is not true. And suppose it is not true; then it is true.

Suppose next that I write on the board (in room 101 this time) the following denoting expressions:

(A) the ratio of the circumference of a circle to its diameter.

(B) the positive square root of 36.

(C) the sum of the numbers denoted by expressions on the board in room 101.

It is clear what the denotation of (A) and (B) are. But what is the denotation of (C)? Suppose C denotes k. Then the sum of the numbers denoted by expressions on the board in room 213 is π+6+k. So C denotes π+6+k. So k=π+6+k. And we are landed in paradox again.

Finally, suppose I write on the board (in room 13) these two predicates:

(E) moon of the Earth

(F) unit extension of a predicate on the board in room 13

(where a unit extension is an extension with exactly one member). The extension of predicate (E) is a unit extension (whose sole member is the Moon). So the extension of (E) is a member of the extension of (F). What about the extension of (F)? Suppose first that it is a self-member. Then the extension of (F) has two members, so it is not a unit extension - and so it is not a self-member. Suppose second that it is not a self-member. Then the extension of (F) has just one member, so it is a unit extension - and so it is a self-member. Either way we obtain a contradiction, and we are landed once again in paradox.

The first of these paradoxes is one version of the Liar paradox, which turns on the notion of truth. The second is related to the so-called paradoxes of definability, like Richard's, König's, and Berry's, which turn on the notion of reference or denotation.[1] The third is related to Russell's paradox. It is natural to suppose (as Frege did) that every predicate has an extension - for example, the predicate 'natural number' has an extension under which just the natural numbers fall, and the predicate 'moon of the Earth' has an extension under which just the Moon falls. But if we suppose (F) has an extension, a paradox arises. And we can generate a version of Russell's paradox if we suppose that the predicate 'non-self-membered extension' has an extension.

II. Reasoning past pathology

Consider again our reasoning about the Liar sentence (L). If we suppose that (L) is true we reach a contradiction; and if we suppose that (L) is untrue we reach a contradiction. So we can conclude that (L) is semantically pathological. Semantic pathologicality may be cashed out in a variety of ways - perhaps (L) is gappy, or ungrounded, or unstable. But if (L) is pathological, then it is not true. That is, we may conclude:

(L*) (L) is not true.

And while (L) is pathological, (L*) is true - even though (L*) is composed of the very same words as (L) with the same linguistic meaning.

Similarly, we can conclude that (C) is pathological - although it appears to be a denoting expression, it does not denote a number, on pain of contradiction. But now we can reason that (A) and (B) are the only expressions on the board that denote numbers. So we may conclude that *the sum of the numbers denoted by expressions on the board in room 101* is π+6. Now in the previous sentence there occurs a token of the same type as (C), call it (C*). And while (C) is pathological, (C*) is an unproblematic denoting expression that denotes π+6.

We also found that (F) is a pathological predicate - it does not have an extension, on pain of contradiction. But if (F) does not have an extension, then in particular it does not have a unit extension. So the only *unit extension of a predicate on the board in room 13* is the extension of (E). We've just produced a token of the same type as (F), call it (F*). But unlike (F), (F*) has a well-determined extension (whose only member is the extension of (E)).

In each of these cases, we produce an unproblematic token of the same type as our pathological token. In so doing, we reason past the pathology of (L), (C) and (F), and reach some definite conclusions about the semantical status of (L), about the numbers that are denoted by expressions on the board, and about the extensions of the predicates on the board. How can we represent this kind of reasoning?

III. A contextual approach to paradox

The tokens (L) and (L*) are composed of the very same words with the same linguistic meaning, and yet (L) is pathological while (L*) is true. There is a change in truth-value without a change in meaning. Similarly with (C) and (C*), and (F) and (F*): there are changes in denotation and extension respectively, and yet no change in meaning. Shifts in truth value, denotation or extension without any shift in meaning suggests some pragmatic difference. And in the course of our stretches of reasoning about (L), (C) and (F), we can discern shifts in a number of contextual parameters.

Let (t) stand for any of our three pathological tokens (L), (C) or (F), and let (t*) stand for (L*), (C*) or (F*). Between the contexts of (t) and (t*), there are differences of time and place and perhaps speaker too. A fourth difference is that (t) and (t*) are embedded in two different stages of the discourse. At the first stage I produce (t), and we argue to the subconclusion that t is pathological; at the second stage, we produce the unproblematic token (t*) on the assumption that (t) is pathological. In general, the correct interpretation of an expression or a stretch of discourse may depend on the larger discourse in which it is embedded. The second stage starts out from a subconclusion that (t) is pathological, established by the first stage of the argument. We can think of the second stage as *reflective* with respect to the first: at the second stage of the reasoning, we reflect on (t)'s pathology. The reflective stage of the reasoning is second not merely in temporal order, but in logical order as well. This logical order constitutes a difference in the relation that each stage of the discourse bears to the discourse as a whole.

A fifth difference is found in speaker's intentions. At the two stages of the reasoning, there are different intentions towards (t). At the second stage, our intention is to treat (t) as pathological and see where that leads us. But when I first produced (t), I had no intention of producing a pathological predicate: my intention was to pick out a sentence (or denoting expressions, or predicates) written on the board next door.

There is another shift between the two stages, a shift of relevant information. The pathological nature of (t) is established only at the completion of the first stage. But this information is available throughout the reflective second stage of the reasoning. The reasoning of the second stage should be interpreted as incorporating this information.

So we distinguish two contexts: one in which we reason to (t)'s pathology, and the second in which we produce (t*). Call these *the initial context* and *the reflective context* respectively. Between these contexts there is a shift in a number of contextual parameters, shifts in speaker, time, place, discourse position, intention, and relevant information. Recall what we want to explain. Predicates (t) and (t*) are two tokens of the same type, one occurring in the initial context, the other occurring in the reflective context. Though these tokens are composed of the same words with the same meaning, one is pathological and the other is not. A pragmatic explanation is indicated, one that takes account of the shift in the contextual parameters. If we accept the appropriateness of a pragmatic explanation, then we should expect to find a term occurring in the two tokens that is context-sensitive.

I propose that, in the absence of any reasonable alternatives, we explore the idea that the context-sensitive terms in (L), (C) and (F) are 'true', 'denotes' and 'extension' respectively. Let 'true$_i$' abbreviate 'true in the initial context', and let 'true$_r$' abbreviate 'true in the reflective context'. Similarly, 'denotes$_i$' and 'denotes$_r$', and 'extension$_i$' and 'extension$_r$'.[2] We take the occurrence of 'true' in (L) to be sensitive to the context in which it occurs. Accordingly (L) is represented as:

(L) (L) is not true$_i$.

Similarly, (C) is represented by

(C) the sum of the numbers denoted$_i$ by expressions on the board in room 101, and (F) is represented by:

(F) unit $extension_i$ of a predicate on the board.

Now in general we give truth conditions for a sentence via the schema:

s is true iff p,

where p abbreviates a sentence, and s is a name of the sentence. Corresponding to the initial context is the i-schema:

s is $true_i$ iff p.

When we try to give (L) truth conditions via the i-schema we are landed in contradiction:

(L) is $true_i$ iff (L) is not $true_i$.

According to our contextual analysis, this is what is happening at the first stage of the Liar reasoning: we reach a contradiction because we try to provide truth-conditions for (L) via the i-schema. We find that (L) is pathological, and so not true in its context of utterance. That is, we infer that (L) is not $true_i$ - which is just our conclusion (L*).

In producing (L*) here, we have in effect repeated (L). But we have repeated (L) in a new reflective context, in which we no longer provide truth-conditions via the i-schema. Instead, we provide truth-conditions via the r-schema:

s is $true_r$ iff p.

And (L*) does have $truth_r$ conditions. Consider the instance:

(L*) is $true_r$ iff (L) is not $true_i$.

Since (L) is not $true_i$, we may infer that (L*) is $true_r$.

(L) and (L*) are semantically indistinguishable. The difference between them is purely pragmatic. It is a matter of the schema by which they are evaluated. At the first stage of the reasoning, (L) is evaluated via the i-schema; at the second stage of the reasoning, (L*) is evaluated by the r-schema. When I first produce the token (L), I say that (L) is not true_i. So to evaluate my utterance, we need to determine whether or not (L) is true_i - that is, we must evaluate (L) via the i-schema. It is the i-schema that is implicated here. We find that we cannot evaluate (L) by the i-schema. At the second stage of the reasoning, we repeat the words of (L), producing the token (L*) in a new reflective context. To evaluate (L*), we use a schema that is sensitive to (L)'s pathology - and here the implicated schema is the reflective r-schema. That the r-schema is reflective in this way is a product of the reasoning of the first stage, the assessment of (L) as pathological, and the intention to treat (L) as such. With the shift in context, there is a shift in the implicated schema.

Notice that if we evaluate (L) by the r-schema, we find that (L), like (L*), is true_r; and if we determine the extension of (L*) by the i-schema, we find that (L*), like (L), does not have truth_i conditions. Neither (L) nor (L*) is in the extension of 'true_i'; both are in the extension of 'true_r'. So 'true' is a context-sensitive term that shifts its extension according to context.

It is a similar story for (C). The denotation of a term is determined by the denotation schema:

s denotes n iff p=n,

where instances of the schema are obtained by substituting for 'p' any referring expression, for 's' any name of this expression, and for 'n' any name of an object. (C) is analyzed as

(C) the sum of the numbers denoted_i by expressions on the board in room 101.

At the first stage of the reasoning, we try to determine the denotation of (C) via the i-schema s denotes_i n iff p=n,

and we obtain a contradiction. We conclude that (C) does not denote_i. So the only expressions on the board that denote_i are (A) and (B). It follows that the sum of the numbers denoted_i by expressions on the board in room 101 is π+6. Here we produce (C*), in effect repeating (C). But (C*) is produced in the reflective context, where the implicated schema is the r-schema:

s denotes_r n iff p=n.

Instantiating this schema to (C*), we obtain

> (C*) denotes_r k iff the sum of the numbers denoted_i by expressions on the board in room 101 is identical to k.

The right hand side is true for k=π+6. So (C*) denotes_r π+6.

The contextual account for (F) runs parallel. The extension-schema is:

x is in the extension of 'φ' iff x is φ,

where φ stands for a predicate. (F) is represented by

(F) unit extension_i of a predicate on the board in room 13.

And we obtain a contradiction when we try to determine an extension for F via the i-schema. We conclude that (F) does not have an extension_i. So we go on to infer that the only unit extension_i of a predicate on the board in room 13 is the extension of (1). Here we produce (F*) in the reflective context, in which extensions are determined by the r-schema:

x is in the extension_r of 'φ' iff x is φ

Instantiating to (F*), we obtain:

> x is in the extension_r of (F*) iff x is a unit extension_i of a predicate on the board in room 13.

The right hand side is true when we put the extension_i of (1) for x, and false otherwise. And so the extension_i of (1) is the sole member of the extension_r of (F*).

In each case, we land in contradiction when we apply the i-schema to the token (t). We conclude that (t) is pathological. Reflecting on (t)'s pathology, we go on to produce (t*) in a new reflective context. And (t*) is provided a truth value or a denotation or an extension by the r-schema. So (L) and (L*) are not true_i, (C) and (C*) do not denote_i, and (F) and (F*) do not have an extension_i. But (L) and (L*) are true_r, (C) and (C*) denote_r, and (F) and (F*) have an extension_r. According to our contextual analysis, then, the expressions 'true', 'denotes', 'extension' are context-sensitive terms that shift their extensions according to context.

IV. Rejecting the hierarchy

The question naturally arises: What is the relation between different occurrences of the term 'true' (or 'denotes' or 'extension')? More specifically, what is the relation between 'true_i' and 'true_r', 'denotes_i' and 'denotes_r', and 'extension_i' and 'extension_r'?

A possible response here is a Tarskian one: when we move from the first stage of the reasoning to the second, we push up a level of language.[3] So the expressions 'true_i' and 'true_r', for example, belong to distinct languages. On such a hierarchical account, the extension of 'true_r' properly contains the extension of 'true_i'. But there are a number of serious difficulties with the Tarskian approach.

Consider how such an account might go. Take the case of 'true', though the account for 'denotes' or 'extension' will be very similar. We start with a fragment of English free of 'true'. This will be the language at the first level, call it L_0. At the next level of language L_1, we can talk about the truths of L_0: L_1 contains the predicate 'true in L_0', applying just to the truths of L_0. In general, $L_{\sigma+1}$ contains the predicate 'true$_\sigma$', which applies exactly to the truths of L_σ.[4]

An immediate worry with this Tarskian account is its artificiality: why suppose that a natural language like English is regimented and stratified in this way? But there are other problems too. My utterance "Snow is white" is of level 1; your utterance of "'Snow is white' is true" is of level 2; and so on through the levels. Your use of 'true' has in its extension all sentences of level 1, *and no others*. So all sentences of level 1 and beyond are excluded from the extension of your use of 'true'. Gödel remarks of Russell's type theory that "...each concept is significant only ... for an infinitely small portion of all objects."[5] A similar point can be made here about a hierarchical account of truth: an ordinary use of 'true' will apply only to a fraction of all the truths.

Further, it is hard to see how levels can be assigned in a systematic way. How are we to interpret a given sentence containing 'true'? To which language does it belong? Except in very simple cases, we will have little basis for an assignment of one level rather than another. And what level should we assign to a global statement like 'Every sentence is either true or not true'? Any assignment of a level here will compromise the global nature of the statement.[6]

These are difficulties for any hierarchical account of truth. And they carry over to hierarchical accounts of denotation and extension. But even if we set aside these difficulties, still we could not hope to provide a unified account of 'true', 'denotation' and 'extension' along hierarchical lines. This is because we cannot give a hierarchical account of extensions. Let us see why.

Extensions can be self-membered. For example, the extension of the predicate 'infinite extension' belongs to itself, because there are infinitely many infinite extensions; more modestly, the extension of 'extension with more than one member' belongs to itself.[7]

Now consider a hierarchical account of extensions. We take L_0 be a fragment of English free of the term 'extension'. In L_1 we can talk about the extensions of predicates of L_0. That is L_1 will contain the expression 'extension of a predicate of L_0', or 'extension$_0$' for short. In general, $L_{\sigma+1}$ contains the expression 'extension$_\sigma$', which applies exactly to the predicates of L_σ.

Russell's paradox is now avoided. Consider the predicate 'non-self-membered extension$_\sigma$' where the subscript indicates level.[8] This applies only to the predicates of L_σ, and the predicate itself is a predicate of $L_{\sigma+1}$, not of L_σ. And so the question of its own extension - whether or not it is a self-member - does not arise.

But now return to the self-membered extensions. Consider, for example, the extension of the predicate 'extension with more than one member'. According to the hierarchical approach, this predicate belongs to some level of language, and is analyzed as 'extension$_\sigma$ with more than one member' - or in full, 'extension of a predicate of L_σ with more than one member'. And the predicate itself is a predicate of $L_{\sigma+1}$ and not of L_σ. So the extension of this predicate is not a self-member - it contains extensions of predicates of L_σ only. The hierarchical account cannot accommodate self-membered extensions. A distinctive feature of extensions is regimented away.

V. A singularity proposal

I am after a unified account of 'true', 'denotes' and 'extension', one that avoids counterintuitive restrictions. The account I shall offer is in a strong sense anti-hierarchical. The leading idea is that occurrences of 'true', 'denotes' and 'extension' are *minimally* restricted. We have seen that the occurrences of 'true' in (L), 'denotes' in (C), and 'extension' in (F) must be restricted: these occurrences do not apply to (L), (C) and (R) respectively. Are there other restrictions, and if so, what are they?

At this point, a pragmatic principle of interpretation comes into play: the principle of *Minimality*. According to Minimality, restrictions on occurrences of 'true', 'denotes' and 'extension' are kept to a minimum: we are to restrict their applications only when there is reason to do so.

Suppose you say: "'Snow is white' is true". Here, your use of 'true' is quite unproblematic. Should (L) be excluded from its extension? Minimality says no. And this is surely plausible. As we've seen, *within* its context of utterance, (L) is pathological; but when it is evaluated outside its context via the r-schema, (L) is true (that is, $true_r$). Now the context of your utterance is neutral with respect to (L) - we have no reason to interpret your utterance as in some way pathologically linked to (L). Consider the n-schema, associated with your neutral context of utterance:

$$s \text{ is } true_n \text{ iff } p.$$

Instantiating to (L), we obtain

$$(L) \text{ is } true_n \text{ iff } (L) \text{ is not } true_i.$$

Since (L) is indeed not $true_i$, it follows that (L) is $true_n$. We do not withhold (L) from the extension of 'true' in your utterance. It would be a poor interpretation that implicated your utterance in semantic pathology. In general, speakers do not usually aim to produce pathological utterances, or utterances implicated in paradox.[9] By adopting Minimality, we respect this pragmatic fact.

Further, Minimality keeps surprise to a minimum. We do expect any solution to a genuine paradox to require some revision of our intuitions. But the more a solution conflicts with our intuitions, the less plausible that solution will be. By Minimality, our uses of 'true' apply to almost all the truths. We are sometimes forced to restrict 'true' - we must, for example, limit the extension of 'true' in (L) by excluding (L) itself. Still, according to Minimality, we are required to exclude only those (pathological) truths that cannot be included.

Similarly, we must exclude (C) from the scope of 'denotes' in (C), and (F) from the scope of 'extension' in (F). But by Minimality a neutral use of 'denotes' will apply to (C), and a neutral use of 'extension' will apply to (F). Restrictions are kept to a minimum.

So my proposal identifies what I shall call *singularities* of 'true', 'denotes' and 'extension'. Given a context α, a sentence token that cannot be given truth conditions via the α-schema is a *singularity* of 'true_α'. And further if the α-schema is the implicated schema for the sentence token in its context, then the token is *pathological*. (L) and (L*) are singularities of 'true_i'. And (L) is pathological, since in its context, it is evaluated by the i-schema. (L*) is *not* pathological; in its context, its evaluating schema is the r-schema, and (L*) is true_r. Similarly, (C) and (C*) are singularities of 'denotes_i', since neither denotes_i. Further, (C) is pathological, because its implicated schema is the i-schema. (C*) is not pathological, because its implicated schema is the r-schema, and (C*) does denote_r. And (F) and (F*) are singularities of 'extension_i', and (F) is pathological, but (F*) is not.

(L), (C) and (F) are singularities only in a context-relative way. There are reflective (and neutral) contexts in which 'true' applies to (L), 'denotes' applies to (C), and 'extension' applies to (F).

No occurrence of 'true', 'denotes' or 'extension' is without singularities. Suppose again that you say "'2+2-4' is true", but then you perversely add "But this very sentence isn't." Because of the anaphoric construction here, your pathological addition shares its context of utterance with your original utterance. And so your pathological addition is a singularity of the occurrence of 'true' in your utterance. It may be that there are no *actual* statements that force restrictions on a given occurrence of 'true'; there may be no *actual* singularities. But perverse additions like yours are always possible, and they will be singularities of the given occurrence of 'true'. We can also construct singularities like these for any given occurrence of 'denotes' or 'extension'.[10]

We are now in a position to see the anti-Tarskian nature of the singularity proposal. As a consequence of Minimality, the singularity proposal is not hierarchical. Consider the case of truth. According to our contextual analysis,

(R) (L) is true_r.

Now, by Minimality, (R) is not excluded from the extension of 'true_i', since (R) is not identified as a singularity of 'true_i'. So in the extension of 'true_i' there is a sentence in which the predicate 'true_r' appears. For the Tarskian, this would amount to an unacceptable mixing of language levels. According to the singularity proposal, there are no such levels. Similarly, there are denoting phrases containing 'denotes_r' within the scope of 'denotes_i', and there are predicates containing 'extension_r' within the scope of 'extension_i'.[11]

Moreover, there are singularities of 'true_r' that are <u>not</u> singularities of 'true_i'. We may add to (R) the words "but this very sentence isn't", and produce anaphorically a singularity of 'true_r'. But by Minimality this is not a singularity of 'true_i'. According to a hierarchical treatment, the extension of 'true_i' is properly included in the extension of 'true_r'. On the singularity account, neither extension includes the other. Similarly, there are singularities of 'denotes_r' that are not singularities of 'denotes$_i$', and singularities of 'extension$_r$' that are not singularities of 'extension$_i$'.[12]

So according to the present proposal, our three paradoxes are to be treated by the identification and exclusion of singularities. We treat everyday English not as a hierarchy of languages, but as a single language. We do not divide up the terms 'true', 'denotes' and 'extension' between infinitely many languages; rather we identify singularities of univocal, context-sensitive terms.

Gödel noted that Russell's theory brings in a new idea for the solution of the paradoxes:

> It consists in blaming the paradoxes ... on the assumption that every concept gives a meaningful proposition, if asserted for any arbitrary object or objects as arguments.[13]

Gödel goes on to say that the simple theory of types carries through this idea on the basis of a further restrictive principle, by which objects are grouped into mutually exclusive ranges of significance, or types, arranged in a hierarchy.

Gödel suggests that we reject this principle, while retaining the idea that not every concept gives a meaningful proposition for any object as argument:

> It is not impossible that the idea of limited ranges of significance could be carried out without the above restrictive principle. It might even turn out that it is possible to assume every concept to be significant everywhere except for certain 'singular points' or 'limiting points', so that the paradoxes would appear as something analogous to dividing by zero. Such a system would be most satisfying in the following respect: our logical intuitions would then remain correct up to certain minor corrections, i.e. they could then be considered to give an essentially correct, only somewhat 'blurred', picture of the real state of affairs.[14]

I take my singularity proposal to be very much in the spirit of Gödel's remarks.

VI. Circularity and singularities

So far, I have spoken informally of pathology and singularities, largely by way of examples. Let me now sketch the beginnings of a more formal treatment of these notions.

Consider again our paradoxes. Why do (L), (C) and (F) lead to trouble? In trying to determine a truth value for (L), we are directed to the sentence written on the board - but that sentence is (L). And in trying to determine a denotation for (C), we are directed to denoting expressions on the board - and one of these is (C). And to determine an extension for (F), we are directed back to (F). Part of the story, then, appears to be a kind of circularity. We might think in terms of 'dependency' trees:

```
(L)                (C)                   (F)
 |               /  |  \                /    \
(L)           (A)  (B)  (C)           (E)    (F)
                       /  |   .              /    .
                    (A)  (B)    .          (E)      .
                                  .                   .
```

where higher entries depend for their evaluation on lower entries. The circularity of (L), (C) and (F) is indicated by the infinite branches on which they repeat.

To make these ideas more rigorous, we need a precise representation of (L), (C) and (F). Suppose we represent (L) as the ordered pair <type(L), true_i>, where the first element is the sentence-type of (L), and the second indicates the appropriate representation of the occurrence of 'true' in (L). Then something is missing. This representation does not distinguish (L) from (L*), yet the former token is pathological and the latter is not. There is something more to consider: the implicated schema which determines the token's truth value in its context. The implicated schema for (L) is the i-schema (and for (L*) the r-schema). So we can represent (L) more perspicuously as an ordered triple - <type(L), true$_i$, true$_i$> - where the third element indicates the schema which determines a truth value for (L) in its context of utterance. Let us call this triple the *primary representation* of (L). The primary representation of (L*) is the triple <type(L), ext$_i$, ext$_r$>, where the third element indicates that in its context, (L*) is evaluated via the r-schema. The primary representation of (C) is <type(C), denotes$_i$, denotes$_i$>, and of (C*), <type(C), denotes$_i$, denotes$_r$>. The primary representations of (F) and (F*) are <type(F), extension$_i$, extension$_i$> and <type(F), extension$_i$, extension$_r$> respectively.

We can also evaluate (L), (C) and (F) from contexts other than their own. For example, we can assess (L) from the subsequent reflective context. We may represent such an assessment as the triple <type(L), true$_i$, true$_r$>. We can think of this as a *secondary representation* of (L), since here the evaluating schema is not the implicated schema. Notice that this secondary representation of (L) is identical to the primary representation of (L*). This is appropriate, since both (L) and (L*) are true$_r$.

Now, with our dependency trees in mind, let us introduce the notion of a *determinant*. When we try to determine a truth value for (L), we are referred to a certain sentence on the board - call this sentence the *determinant* of (L). In the case of (C), we are referred to certain denoting expressions, and these are (C)'s determinants. And (F)'s determinants are the predicates to which (F) makes reference.

Notice that the occurrence of 'true' in (L) is represented by 'true$_i$'. So in order to evaluate (L), we must determine whether or not the sentence on the board is true$_i$. That is, the schema implicated for (L)'s determinant is the i-schema. Similarly, since 'denotes$_i$' occurs in (C) and 'extension$_i$' occurs in (F), the schemas implicated for their determinants are the respective i-schemas.

We are now in a position to introduce the notion of a *primary tree*. The primary tree for (L) looks like this:

$$
\begin{array}{c}
\langle \text{type(L), true}_i\text{, true}_i \rangle \\
| \\
\langle \text{type(L), true}_i\text{, true}_i \rangle \\
. \\
. \\
.
\end{array}
$$

To construct the primary tree for (L), we start with the primary representation of (L). This is the node at the top of the tree. At the second tier is the determinant of (L), namely (L), suitably represented: since 'true$_i$' occurs in (L), its determinant (L) is to be evaluated via the i-schema. Acordingly we represent (L) at the second tier as $\langle$type(L), true$_i$, true$_i\rangle$. This is the primary representation of (L) again, which will repeat at the third node. And so on, indefinitely.

The primary tree for (L) is composed of a single infinite branch, on which the primary representation of (L) repeats. This indicates that (L) is pathological. The repetition of its primary representation shows that a truth value for (L) cannot be determined by the i-schema - and so we can also say that (L) is a singularity of 'true$_i$'.

The primary tree for (C) is:

$$
\begin{array}{ccccc}
 & \langle \text{type(C), denotes}_i\text{, denotes}_i \rangle & & & \\
/ & | & \backslash & & \\
\text{type(A)} & \text{type(B)} & \langle \text{type(C), denotes}_i\text{, denotes}_i \rangle & & \\
 & / & | & . & \\
 & \text{type(A)} & \text{type(B)} & & . \\
 & & & & \quad .
\end{array}
$$

Notice that (A) and (B) contain no context-sensitive terms, and so they are adequately represented by their types. Notice also that the occurrence of 'denotes' in (C) is represented by 'denotes$_i$', and so the implicated schema for (C)'s determinants is the i-schema. (C) is a determinant, and so at the second tier its appropriate representation is <type(C), denotes$_i$, denotes$_i$>. This is the primary representation of (C) again, which in turn generates a third tier of nodes. And so on, indefinitely. So (C)'s primary tree contains an infinite branch on which the primary representation of (C) repeats. This indicates that (C) is pathological and a singularity of 'denotes$_i$'.

The primary tree for (F) is:

```
                <type(F), extension_i, extension_i>
               /                                  \
          type(E)                   <type(F), extension_i, extension_i>
                                     /                              .
                               type(E)                                .
                                                                        .
```

This tree has an infinite branch on which the primary representation of (F) repeats. (F) is pathological, and a singularity of 'extension$_i$'.

Consider now (L*). Since 'true$_i$' occurs in (L*), and (L*)'s implicated schema is the r-schema, the primary representation of (L*) is the triple <type(L), true$_i$, true$_r$>. (L*) makes reference to the sentence written on the board, so its determinant is (L). Since 'true$_i$' occurs in (L*), the appropriate representation of this determinant is <type(L), true$_i$, true$_i$>, and this constitutes the second node of (L*)'s primary tree. This is in turn generates a third node, <type(L), true$_i$, true$_i$> again. And so on. The primary tree for (L*) is:

<type(L), true$_i$, true$_r$>
|
<type(L), true$_i$, true$_i$>
|
<type(L), true$_i$, true$_i$>
.
.
.

The primary representation of (L) repeats on this single infinite branch, but the primary representation of (L*) does not. (L*)'s primary representation stands above the circle in which (L)'s primary representation is caught. (L)'s circularity is not the end of the matter: we can reason past pathology.[15] We can give expression to this idea via the notion of a <u>pruned</u> tree. We <u>prune</u> the tree here by terminating the infinite branch at the first occurrence of a non-repeating node. The pruned tree is simply:

<type(L), true$_i$, true$_r$>.

This wellfounded tree indicates that (L*) is not circular. We can determine a truth value for (L*) via the r-schema. (L*) is not pathological, and it is not a singularity of 'true$_r$'.

In contrast, we cannot prune the primary tree for (L), because there are no non-repeating nodes on its infinite branch. (L) cannot be evaluated by the i-schema. However, as we've seen, (L) can be evaluated by the reflective r-schema. And this is indicated by our trees. Consider this secondary representation of (L): <type(L), true$_i$, true$_r$>. This generates what we will call a *secondary* tree for (L), identical to the primary tree for (L*). We obtain the same wellfounded pruned tree, which indicates that we can determine a truth value for (L) via the r-schema: (L) is not a singularity of 'true$_r$'. Further, suppose we evaluate (L) in some neutral context. Let the associated secondary representation of (L) be <type(L), true$_i$, true$_n$>. The corresponding secondary tree will be exactly like (L*)'s primary tree (with the subscript 'n' replacing the subscript 'r'). Again, we obtain a wellfounded pruned tree: (L) is not a singularity of 'true$_n$'. All this accords with Minimality. We do not place restrictions on occurrences of 'true' unless we have to.

The primary tree for (C*) is:

$$
\begin{array}{l}
\qquad\qquad \langle \text{type(C), denotes}_i\text{, denotes}_r \rangle \\
\qquad\quad / \qquad\qquad | \qquad\qquad\qquad \backslash \\
\text{type(A)} \qquad\quad \text{type(B)} \qquad\quad \langle \text{type(C), denotes}_i\text{, denotes}_i \rangle \\
\qquad\qquad\qquad\qquad\qquad\qquad / \qquad\quad | \qquad\quad \backslash \\
\qquad\qquad\qquad\qquad\quad \text{type(A)} \quad \text{type(B)} \quad \langle \text{type(C), denotes}_i\text{, denotes}_i \rangle. \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad / \qquad\quad | \qquad\qquad . \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{type(A)} \qquad \text{type(B)} \qquad\quad . \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad .
\end{array}
$$

The pruned tree is:

$$
\begin{array}{l}
\langle \text{type(C), denotes}_i\text{, denotes}_r \rangle \\
\quad / \qquad\qquad\qquad\qquad \backslash \\
\text{type(A)} \qquad\qquad\quad \text{type(B)}
\end{array}
$$

The infinite branch on the primary tree indicates that (C) is pathological. When we prune the tree, we eliminate (C) as a determinant - this is just what we do at the reflective stage of the reasoning. The wellfounded pruned tree indicates that (C*) denotes$_r$, and that its denotation$_r$ depends only on (A) and (B).

The account for (F*) runs parallel, and we arrive at this pruned tree:

$$
\begin{array}{c}
\langle \text{type(F), extension}_i\text{, extension}_r \rangle \\
| \\
\text{type(E)}
\end{array}
$$

The pruned tree is wellfounded, and it indicates that (F*) does have an $extension_r$ depending only on the predicate (E).

We can give a more general characterization of the notions of *circularity* and *singularities*. We have been dealing with sentence tokens containing 'true', denoting tokens containing 'denotes', and predicate tokens containing 'extension'. Let t be any such token. Either 'true' or 'denotes' or 'extension' occurs in t; suppose this occurrence is represented by '$true_{c\alpha}$' or '$denotes_{c\alpha}$' or '$extension_{c\alpha}$'. And suppose the implicated schema is the c_β-schema. Let the primary representation of t be given by $<type(t),c_\alpha,c_\beta>$. Take t's primary tree, and prune it.[16] If t's primary representation repeats on an infinite branch of t's pruned tree then t is *circular,* and t is a *singularity* of '$true_{c\beta}$' or '$denotes_{c\beta}$' or '$extension_{c\beta}$'.

VII. Universality

Natural languages are remarkably flexible and open-ended. If there is something that can be said, it might seem that a natural language like English has at least the potential to say it. Natural languages evolve; they always admit of expansion, of increased expressive power. Tarski speaks of the "all-comprehensive, universal character" of natural language, and continues:

> The common language is universal and is intended to be so. It is supposed to provide adequate facilities for expressing everything that can be expressed at all, in any language whatsoever... .[17]

I think that the singularity proposal goes a long way to accommodate this intuition. Occurrences of the context-sensitive terms 'true', 'denotes' and 'extension' are as close to global as they can be without contradiction - they apply everywhere except to their singularities. Moreover, according to the singularity proposal, even sentences that are singularities of 'true' relative to a given context are in the scope of 'true' in other contexts (such as an appropriate reflective context or a neutral context) - and similarly for singularities of 'denotes' and 'extension'. So the singularities that prevent uses of 'true' or 'denotes' or 'extension' from being fully global are captured by other uses of these expressions.

We can take these points a little further. Many have thought that the goal of a universal language is unattainable, because any theory of the semantical and logical terms of a language must be couched in an essentially richer metalanguage.[18] But if we adopt the singularity proposal, then we are not driven to this conclusion. Let L be that fragment of English that is free of context-sensitive terms. Let $L^{/}$ be the result of adding to L the context-sensitive term 'true' or 'denotes' or 'extension'. We can think of $L^{/}$ as the object language for our singularity account. Now in this paper I have not attempted to give a formal theory of 'true' or 'denotes' or 'extension'. I have only described some notions (like *primary tree*, *pruned tree*, *circularity*, and *singularity*) that I take to be central to such a theory. But suppose for a moment that we have a formal singularity theory.[19]

With any theory of context-sensitive terms, there is an inevitable separation of the object language and the language of the theory. We can take the language of the theory to be a classical formal language which quantifies over contexts, and in which context-sensitive terms do not appear. Unlike the object language, the language of the theory is context-independent. Now the language of the theory - call it T - will be in certain ways richer than the object language $L^{/}$. For example, in the case of 'true', T will contain the predicate "sentence of $L^{/}$ that is true in some context" - call this predicate 'true_{obj}'. Among the sentences to which 'true_{obj}' applies are all the singularities that are true in some context. Given that any occurrence of the context-sensitive term 'true' has singularities, 'true_{obj}' will be in this way more inclusive than any occurrence of 'true'. Parallel remarks can be made about '$\text{denotes}_{\text{obj}}$' ('expression of $L^{/}$ that denotes in some context') and '$\text{extension}_{\text{obj}}$' ('predicate of $L^{/}$ that has an extension in some context').

This may tempt us to suppose that 'true_{obj}' ('$\text{denotes}_{\text{obj}}$', '$\text{extension}_{\text{obj}}$') is more comprehensive than any occurrence of 'true' ('denotes', 'extension'), and that T is a Tarskian metalanguage for $L^{/}$. But the temptation should be resisted. Let us see why.

Consider a sentence of T containing 'true_{obj}' - say, "(L) is true_{obj}". Though such a sentence is not a sentence of _$^{/}$, it will not be excluded from the extension of a context-sensitive occurrence of 'true' in $L^{/}$. Only the singularities of the given occurrence are excluded - and non-pathological sentences expressed in T will not be identified as singularities. (Intuitively, the theory tells us what is *excluded from* the scope of occurrences of 'true', *not* what is included; we take a 'downward' route rather than an 'upward' route.) This shows that T is not a Tarskian metalanguage for $L^{/}$, since ordinary context-sensitive uses of 'true' apply to sentences that are expressible in T but not in _$^{/}$. Again, according to the singularity proposal, paradox is avoided not by a Tarskian ascent, but by the identification and exclusion of singularities. (Similarly, denoting expressions of T containing '$\text{denotes}_{\text{obj}}$' are in the scope of 'denotes'; and predicates of T containing '$\text{extension}_{\text{obj}}$' are in the scope of 'extension'.)

Moreover, if we suppose that T is a classical formal language, free of context-sensitive terms, then T cannot contain the expression 'true in T', on pain of contradiction.[20] So we can generate from this formal language a Tarskian hierarchy of formal languages, each containing a truth predicate for the preceding language. But none of the sentences expressible in these languages are identified as singularities, and so none are excluded from the extensions of our ordinary context-sensitive uses of 'true'. To speak metaphorically, our context-sensitive uses of 'true' arch over not only the sentences of T, but also all the sentences expressed by the languages of this hierarchy. Similarly, 'denotes' and 'extension' apply respectively to denoting expressions and predicates of T and the languages of the hierarchy.

So we do not take the formal hierarchy generated from T to explicate 'true' or 'denotes' or 'extension'. The levels do not correspond to any stratification of these context-sensitive terms. The singularity proposal abandons this Tarskian route. For the Tarskian, questions about the extent of the hierarchy and quantification over the levels will present special difficulties. Of course, these are substantial questions, quite independently of any particular proposal about 'true' or 'denotes' or 'extension' in English. But they present no special difficulty for the singularity account. According to the singularity account, an ordinary context-sensitive use of 'true' applies almost everywhere, failing to apply only to the sentences that are pathological in its context of utterance. By Minimality, when we use the term 'true' we point to as many truths as we can point to from our context of utterance. These truths include those expressible in T, and at any level of the Tarskian hierarchy which can be generated from that theoretical language (whatever the extent of the hierarchy). And similarly for 'denotes' and 'extension'.

To return to universality. With respect to the concept of truth, a universal language would have a term applying to <u>all</u> the true sentences of the language. Such a language would be subject to the Liar. According to the singularity account, any use of 'true' has its singularities. But a use of 'true' applies everywhere else - even to sentences couched in the language T of the theory, and the hierarchy of languages generated from T. Indeed, a use of 'true' applies to any sentence of any language, as long as the sentence is not identified as a singularity. If we adopt the singularity proposal, then any use of 'true' - or 'denotes' or 'extension' - is as close to global as it can be. In this way we respect Tarski's intuition that we can say everything there is to say.

VIII. Ramsey's division

In this paper, I have proposed a singularity solution to a version of the Liar, a 'definability' paradox, and a member of the Russell family of paradoxes. According to Ramsey, however, these paradoxes "fall into two fundamentally distinct groups".[xxi] Group A includes Russell's paradox; Group B includes the Liar and definability paradoxes. The paradoxes of Group A "involve only logical or mathematical terms such as class and number, and show that there must be something wrong with our logic or mathematics".[xxii] Ramsey continues:

> "But the contradictions of Group B are not purely logical, and cannot be stated in logical terms alone; for they all contain some reference to thought, language, or symbolism, which are not formal but empirical terms. So they may be due not to faulty logic or mathematics, but to faulty ideas concerning thought and language".[xxiii]

Ramsey's distinction between the 'logical' and the 'semantical' paradoxes has been widely accepted. It is natural to conclude (with Ramsey) that the two kinds of paradox should be resolved along quite different lines.

If we take the singularity approach, we cannot divide up the paradoxes this way. For the singularity solution crosses Ramsey's lines. This may incline us to seek a unified account of the paradoxes, as Russell did.[xxiv] Or we might adjust Ramsey's classification, and allow that there are semantical versions of Russell's paradox. Or we might give up the idea that there is a clear distinction between 'logical' and 'semantical' paradoxes. One way or another, the lines must be redrawn.

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Endnotes

1. See Richard 1905, König 1905, and for Berry's paradox, Russell 1908.

2. For ease of exposition, I use the subscript 'i' to indicate the initial context for each of our cases, though of course the three contexts are quite distinct. Similarly with the subscript 'r'.

3. For contextual accounts that appeal to a hierarchy, see Parsons 1974, Burge 1979, Barwise and Etchemendy 1987, and Gaifman 1988 and 1992.

4. And the Liar is now avoided. For example, if someone says "This sentence is false$_\sigma$", then their statement is of level σ+1, and not a false statement of level σ at all. So what the person says is straightforwardly false. As Russell puts it: "... hence his statement is false, and yet its falsehood does not imply, as that of "I am lying" apeared to do, that he is making a true statement. This solves the Liar." (Russell 1908, p.166)

5. Kurt Gödel 1944, in Schilpp 1944, p.149.

6. For further discussion of these and related difficulties, see Gupta 1982, in Martin 1984, pp.204-5.

7. In contrast, sets are never self-members. Consider Zermelo-Fraenkel set theory (ZF), nowadays the widely received set theory. ZF embodies to a degree a certain conception of set. Think of a set as formed this way: we start with some individuals, and collect them together to form a set. Suppose we start with individuals (non-sets) at the lowest level. (In pure set theory, we start with just the empty set.) At the next level, we form sets of all possible combinations of these individuals. And then we iterate this procedure: at the next level, we form all possible sets of sets and individuals from the first two levels. And so on. This conception of set is the *combinatorial* or *iterative* conception. And on this conception, it is clear that no set can be a self-member.

In sharp contrast, extensions are given through predication. Extensions are always extensions of predicates. And whenever an extension of a predicate has the property to which the predicate refers, we have a self-membered extension. In my view, sets and extensions are independent - neither can be reduced to the other. They embody quite different ways of thinking about collections. We might regard them as primitive notions, or perhaps as alternative, mutually irreducible conceptions of the more general notion of a collection. And they provide two very different settings for Russell's paradox. (For more on sets and extensions, see Simmons 1998.)

8. The proponent of the hierarchical resolution will owe us an explanation of how the level is fixed.

9. Of course, philosophical discussions of paradoxes provide exceptions to this general rule.

10. For example, you may utter the innocent enough denoting phrase "the number denoted by

'the square of 1'", but then perversely continue: "plus the number denoted by 'the square of 2', plus the sum of the numbers denoted by phrases in this utterance." Or suppose you say "the extension of 'natural number'", and continue: "unioned with any non-self-membered extension of a nine-word predicate in utterance U", where you stipulate that utterance U is your very utterance.

11. For example, we concluded that (C*) denotes$_r$ π+6. We might add: "And so the number denoted by (C*) is irrational." This addition contains a denoting phrase which, given the context, is represented by "the number denoted$_r$ by (C*)". This phrase does denote$_i$; it is within the scope of 'denotes$_i$'.

For another example, we concluded that (2*) has a well-determined extension$_r$. Consider the occurrence of the predicate 'has a well-determined extension$_R$' in our conclusion. This predicate token does have an extension$_i$; it is within the scope of 'extension$_i$'.

12. Consider again our conclusion that (C*) denotes$_r$ π+6. We might perversely add: "And so the number denoted by (C*), plus the number denoted by 'the square of 2', plus the sum of the numbers denoted by phrases in this utterance, is irrational." Given the context, the occurrences of 'denotes' in our continuation will be represented by 'denotes$_r$'. The final definite description token in our utterance (beginning 'the sum of') is pathological - and since we cannot give it denotation$_r$ conditions, it is a singularity of 'denotes$_r$'. But, by Minimality, this token is not a singularity of 'denotes$_i$'.

And consider again our conclusion that (2*) has a well-determined extension$_r$. You might perversely add:

> "Now form the union of the well-determined extension of (2*) and any empty extension of a nine-word predicate in utterance V",

where you stipulate that V is your perverse addition. Consider the predicate token - call it w - composed of the last nine words of V. Given the context, w is represented by "empty extension$_r$ of a nine-word predicate in utterance V". It is straightforward to check that w is a singularity of 'extension$_r$' - the r-schema cannot provide an extension for it. It is also straightforward to check that w has an extension$_i$ (in fact, the empty extension, since no nine-word predicate in V has an extension$_r$, empty or otherwise). So w is not a singularity of 'extension$_i$'.

13. Kurt Gödel 1944, in Schilpp 1944, p.228.

14. Op. cit., p.229.

15. Here I differ from Gupta and Belnap. Gupta and Belnap resist the idea that we can break out of the circle. They write:

> "It is possible to give a meaning to 'true' by which the pathological sentences would count as 'untrue'... We do not think, however, that this is the ordinary meaning of 'true'. Any assertion that 'the Liar is untrue', even when made with the full consciousness of the Liar's paradoxicality, invites the response that the Liar must then be true, since it asserts its own untruth. The circle of semantical reflection is not naturally broken at any point." (Gupta

and Belnap 1993, p.255, fn.5)

According to the singularity account, the reasoning that produces (L*) is natural, and does not stretch the meaning of 'true' beyond the ordinary. And we may accept the invitation to respond that (L) is true - that is, $true_r$. Both (L) and (L*) are $true_r$, because (L) asserts of itself that it is not $true_i$, and it isn't. The circle is broken once we produce (L*) in the reflective context.

16. In a more developed account, the characterization of a pruned tree will need some refinement. For example, we cannot simply say that if the primary tree has an infinite branch, then we prune that branch, terminating it at the first non-repeating node. We can say that if the primary tree has any infinite branch on which a node repeats, then we prune that branch, terminating it at the first non-repeating node. We need the qualification "on which a node repeats" because we do not prune infinite branches on which no node repeats. Consider the following example. Suppose that at the Great Rock on Monday, someone says:

(1) The sentence uttered here tomorrow will be true.

And that is all that is said at the Great Rock on Monday. On Tuesday, just one thing is said at the Great Rock, viz., a token of the same type as (1). And so on, ad infinitum. The primary tree for (1) (and for all subsequent sentences) is composed of a single infinite branch, on which no node repeats. Intuitively, (2) is pathological, and its infinite tree indicates that. And so we do not wish to prune the tree, even though no node on its infinite branch repeats.

17. Tarski 1969, p.89. See also Tarski 1983, p.164.

18. See, for example, Kripke 1975, in Martin 1984, p.79 and footnote 34.

19. Such a theory is given for 'true', in Simmons 1993. See Simmons 1994 for a singularity account of Richard's, Berry's and König's definability paradoxes, along the lines suggested by our present treatment of (C).

20. For example, if 'true in T' is a predicate of T, we can form the paradox-producing sentence of T: "This sentence is not true in T".

xxi. Ramsey 1925, p.20.

xxii. ibid.

xxiii. op. cit., pp.20-21.

xxiv. Russell argued that the paradoxes all have something in common: each involves a phrase or assertion that refers to an illegitimate totality. For example, consider the case of Berry's paradox, generated by the phrase: "the least integer definable in fewer than nineteen syllables". This phrase refers to a totality of which it is itself a member, namely the totality of phrases that define numbers in fewer than nineteen syllables. In Russell's view, this totality is illegitimate. Russell writes:

"All our contradictions have in common the assumption of a totality such that, if it were legitimate, it would at once be enlarged by new members defined in terms of itself." (Russell 1908, in van Heijenoort 1967, p.154)

According to Russell, all the paradoxes violate the Vicious Circle Principle: "Whatever involves *all* of a collection must not be one of the collection" (ibid).

Like Russell's account, the singularity proposal explains pathology in terms of circularity, and rejects Ramsey's division. But Russell's positive theory - the ramified theory of types, first presented in Russell 1908 - is hierarchical, and the singularity theory is not.