Chapter 4

Identifying singularities

4.1 Primary trees

My aim in this chapter is to lay out the central notions that will allow us to identify singularities. The notions I shall be presenting here – such as the notions of a *primary representation* and a *primary tree* - will figure in the general, formal theory of singularities to be presented in Chapter 6. But here I will be introducing these notions in the focused setting of the simple paradoxes generated by the expressions C, P and L.

Consider again the expression C, and the *repetition* discourse. How should we represent the token C? We've let 'denotes$_{cC}$' represent the occurrence of 'denotes' in C, and any coextensive occurrence of 'denotes' (in whatever context). So a first thought would be to represent C by the ordered pair <type(C),c_C>, where the first member of the pair indicates the type of C, and the second indicates the appropriate representation of the occurrence of 'denotes', by specifying the subscript. But now consider the token C*. This is also represented by <type(C),c_C>, because the type of C* is identical to the type of C, and the extensions of 'denotes' in C* and 'denotes' in C are identical. So now both C and C* are represented by the same ordered pair, and we've failed to distinguish C and C*. But C is pathological and C* isn't. Something is missing.

There is more to consider: we have to consider the kind of schema by which the token is evaluated. At the second stage of *repetition*, you attempt to evaluate C by the c_C-schema, a schema unreflective with respect to C. So we can represent C by the ordered *triple* $\langle \text{type}(C), c_C, c_C \rangle$, where the third member indicates the schema by which C is assessed at the second stage. Now consider C*. At the third stage of *repetition*, you produce C* in a context that is reflective with respect to C. Now when you produce C*, when you say that the sum of the numbers denoted by expressions on the board is π+6, you don't explicitly evaluate C*. At this stage, there is no *specific* reflective schema at work. An explicit evaluation of C* doesn't happen until the fourth stage -- when you produce E, and specifically employ the reflective c_E-schema. But it's clear that we can only accommodate what you say at the third stage if C* is evaluated by the kind of schema that yields the result π+6, the kind of schema that takes into account C's pathology and its failure to denote$_{cC}$. The context c_{C*} fixes a certain *kind* of schema by which C* is to be assessed – a schema that is reflective with respect to C, or, for short, *an r_C-schema*. Any r_C-schema will yield π+6 as the denotation of C*. So we can represent C* by the triple $\langle \text{type}(C), c_C, r_C \rangle$, where the third element indicates that C*'s denotation is that determined by any r_C-schema. The c_E-schema is an r_C-schema, and in the repetition discourse, the c_E-schema emerges as C*'s evaluating schema. But we can easily imagine the repetition discourse unfolding differently, once C* is produced. An explicit reflective evaluation of C* might be produced by someone else at a different time and place. But though the specific evaluating schema would be different from the c_E-schema, it will be an r_C-schema, and it will yield π+6 as C*'s denotation. So the representation $\langle \text{type}(C), c_C, r_C \rangle$ captures all we want to capture about C*. We now have two distinct representations of C and C*.

We may of course evaluate C and C* by other schemas (for example, as I noted in Chapter 2, C may be assessed by the c_E-schema, and C* by the c_C-schema). But in the course of *repetition*, C is evaluated by the c_C-schema, and it is this evaluation that leads to the conclusion that C is pathological, while C*'s denotation is delivered by any r_C-schema (specifically, at the fourth stage, by the c_E-schema), and it is this that leads to the explicit conclusion that C* has a determinate referent. So if we are after an analysis of the repetition discourse, the representation $\langle \text{type}(C),c_C,c_C \rangle$ of C is privileged over other representations of C, and $\langle \text{type}(C),c_C,r_C \rangle$ is likewise a privileged representation of C*. We will call these representations the *primary representations* of C and C*.

Similarly with P and P*, and L and L*. The primary representations of P is $\langle \text{type}(P),c_P,c_P \rangle$, and the primary representation of P* is $\langle \text{type}(P),c_P,r_P \rangle$, where the third entry indicates that P*'s extension is determined by any r_P-schema – that is, any schema reflective with respect to P. At the fourth stage, a specific r_P-schema is used, namely the c_E-schema. The primary representations of L and L* are respectively $\langle \text{type}(L),c_L,c_L \rangle$ and $\langle \text{type}(L),c_L,r_L \rangle$. An r_L-schema is a schema that is reflective with respect to L, and at the fourth stage, L* is evaluated (as true) by a specific r_L-schema, namely, the c_Σ-schema.

There is an obvious but noteworthy feature of the primary representations of repetitions. Take the case of C*. A little terminology: C* makes reference to the expressions on the board, A, B and C – call these the *determinants* of C*, since C*'s denotation will be determined by their denotations. Consider the primary representation of C*, $\langle \text{type}(C),c_C,r_C \rangle$. The second element indicates that 'denotes$_{c_C}$' represents the occurrence of 'denotes' in C, so C*'s determinants are evaluated by the c_C-schema. So we can say that the second element of C*'s primary representation indicates the schema by which C*'s determinants are evaluated. The third element indicates that C*'s denotation is determined by any r_C-schema. But the c_C-schema is not an r_C-schema. The schemas that determine a denotation for C* are different from the schema that determines the denotation of C*'s determinants.

By way of contrast, consider an unexceptional use of 'denotes'. Suppose that you produce the denoting phrase D:

D. the successor of the number denoted by 'the only even prime'.

Here, D's determinant is the expression 'the only even prime'. We let 'denotes$_{cD}$' represent any use of 'denotes' that is coextensive with the occurrence of 'denotes' in D. So D is represented as "the successor of the number denoted$_{cD}$ by 'the only even prime'". D's determinant is evaluated by the c_D-schema – and then, to evaluate D itself, we need no other schema. To evaluate D, only the c_D-schema is used -- we just figure out that 'the only even prime' denotes$_{cD}$ 2, and add 1. So, since the c_D-schema is the only schema we use to evaluate D, the primary representation of D is <type(D),c_D,c_D>. Here the second and third elements are the same. And this is also true of the primary representation of the pathological expression C: as we've seen, its primary representation is <type(C),c_C,c_C>. The second element indicates that C's determinants (A, B and C, the expressions on the board) are evaluated by the c_C-schema – but C is one of those expressions, so C itself is evaluated by the c_C-schema. In contrast to both D and C, the primary representation of a repetition such as C* will have different second and third elements.

More generally, let σ be an expression token (a denoting expression, a predicate or a sentence) containing a token *t* of the type 'denotes', 'extension' or 'true', and let c_σ be the context of σ. Let's start with the notion of a *representation* of σ. A representation of σ is a triple. The first entry indicates the type of σ. The second entry is c_σ, indicating that *t* is to be represented by '$t_{c\sigma}$' -- a representation that applies not only to *t*, but also to any coextensive occurrence of the type of *t*. The third entry is the context which fixes the evaluating schema for σ. The third entry need not be c_σ: different contexts fix different evaluating schemas, and we can consider the schema fixed by the context c_σ or by some other context. So the expression σ has any number of representations. The *primary representation* of σ shares the same two entries with any representation of σ, but the third entry is c_σ – the evaluating schema is the one fixed by σ's own context. So the primary representation of σ is $<\text{type}(\sigma),c_\sigma,c_\sigma>$.

Think of this as a preliminary characterization of a primary representation. It works well for unexceptional expressions like D – D's primary representation is $<\text{type}(D),c_D,c_D>$. And it works well for pathological expressions like C, whose primary representation is $<\text{type}(C),c_C,c_C>$. But with repetitions we can do better. Suppose that σ is a repetition of a pathological expression ρ. Then σ's primary representation $<\text{type}(\sigma),c_\sigma,c_\sigma>$ can be more informatively presented as $<\text{type}(\rho),c_\rho,r_\rho>$, since type(σ)=type(ρ), the occurrence of *t* in σ is coextensive with the occurrence of *t* in ρ, and the context c_σ fixes an r_ρ-schema (a schema reflective with respect to ρ) as the kind of schema by which σ is to be evaluated.

The notion of a *repetition* is easily broadened. Consider the case of C, and suppose that you have reached the third stage of repetition – you have just reached the conclusion that C is pathological. You could now continue:

> So *the number you get when you add up the numbers denoted by the expressions on the board in room 213* is π+6.

Call the italicized expression C^v. Its primary representation is $<\text{type}(C^v),c_C,r_C>$. Just like C, the occurrence of 'denotes' in C^v is represented by 'denotes_{cC}', and C^v's context fixes an r_C-schema as the kind of schema by which C^v is to be evaluated In terms of its primary representation, C^v

differs from C* only in its type. There are any number of such variants of the repetition C*. Their primary representations will capture what is crucial – the occurrence of ‘denotes’ in a variant is coextensive with the occurrence of ‘denotes’ on C, and the variant’s context fixes an r_C-schema as the appropriate kind of evaluating schema.[1]

To sum up the role of the primary representations: the repetition reasoning reveals the differential status enjoyed by C and C* - C is pathological and C* is not. We wish to capture the difference between C and C* in a precise way, and for that we need representations of C and C* that formally capture our intuitive treatment of these phrases. And only primary representations will do. In particular, it is the primary representation of C that reveals C's pathology - the evaluation of C by the c_C-schema, the schema associated with C's context of utterance, leads to contradiction. And it is the primary representation of C* that reveals C*'s status -- the evaluation of C* by an r_C-schema produces a denotation for C*. We are interested in identifying semantic pathology, and if pathology is there to be found, primary representations will help to reveal it.

A *secondary representation* of a token σ involves the evaluation of σ by a schema or a kind of schema other than that fixed by σ’s context. For example, consider the evaluation of C by an r_C-schema. This evaluation is associated with the representation $<\text{type}(C),c_C,r_C>$ - a secondary representation of C. Notice that this secondary representation of C is identical to the primary representation of C*. This is appropriate, since both C and C* denote$_{rC}$ the same number – any r_C-schema yields π+6 as the denotation of C and of C*. Similarly, the primary representation of C is a secondary representation of C*.

We have said that C is pathological. Let us now represent its pathological character in a rather more rigorous way. Generally speaking, some denoting expressions do not make reference to other denoting expressions; the denoting expression 'the only even prime' is like this.

Other denoting expressions do make reference to other denoting expressions -- for example, the denoting expression "the number denoted by 'the only even prime'". But this denoting expression is unproblematic, because it makes reference to a denoting expression that does not make reference to a denoting expression. We may iterate, and obtain increasingly deeply nested denoting expressions, starting with the denoting expression, 'the number denoted by "the number denoted by 'the only even prime'"'. All denoting expressions in this sequence are unproblematic because ultimately they may be traced back to a denoting expression that does not make reference to denoting expressions. Such denoting expressions are, intuitively, grounded. But other denoting expressions are ungrounded. For example, C makes reference to itself, and so in tracing back through the denoting expressions to which C makes reference, we never escape denoting expressions that make reference to denoting expressions.

In a moment, we will represent the pathology of a denoting expression like C via a certain kind of tree. But we need to prepare the ground a little. Let the *determination set* of a denoting expression be the set of the expressions's determinants. The determination set for C, and for C*, is the set {A,B,C}. (The determination sets of any pathological phrase and a repetition of it will be identical.) In general, a phrase containing 'denotes' will make reference to the denotations of certain denoting phrases, and these phrases are the members of its determination set.

It bears emphasizing that C purports to denote a number in terms of what the members of its determination set denote$_{cC}$, since the occurrence of 'denotes' in C is represented by 'denotes$_{cC}$'. So to determine a value for C (if it has one), we need to determine what numbers are denoted$_{cC}$ by the members of its determination set. Now C is itself a member of its own determination set - so in order to determine the number that C denotes, we need to determine what number is denoted$_{c_C}$ by C (along with the numbers denoted$_{c_C}$ by A and B). But when we try to evaluate C by the c_C-schema, we reach a contradiction. And so we conclude that C is pathological.

We will capture C's pathology by a certain kind of tree - its *primary tree*. To construct the primary denotation tree for C, we start with the primary representation of C, the triple $<\mathrm{type}(C),c_C,c_C>$. This is the node at the top of the tree. At the second tier are the members of C's determination set, suitably represented. A and B contain no context-sensitive terms, and so these are suitably represented via their types - we need not worry about the schema by which they are assessed.[2] This isn't so for the other member of C's determination set, namely C itself. In line with the remarks of the previous paragraph, C is to be evaluated by the c_C-schema. Accordingly, we represent C at the second tier as $<\mathrm{type}(C),c_C,c_C>$. More formally, *the second element of the primary representation is the third element of the triples at the second tier*. So the primary representation of C appears at the second tier, and this in turn generates a third tier of nodes, at which the primary representation again appears. And so on, indefinitely. The primary tree for C looks like this:

$$
\begin{array}{cccccccc}
 & & <\mathrm{type}(C),c_C,c_C> & & & & & \\
 & / & | & \backslash & & & & \\
 & \mathrm{type}(A) & \mathrm{type}(B) & <\mathrm{type}(C),c_C,c_C> & & & & \\
 & & / & | & \backslash & & & \\
 & & \mathrm{type}(A) & \mathrm{type}(B) & <\mathrm{type}(C),c_C,c_C> & & & \\
 & & & / & | & . & & \\
 & & & \mathrm{type}(A) & \mathrm{type}(B) & . & & \\
 & & & & & & . &
\end{array}
$$

This tree has an infinite branch, *on which the primary representation of C repeats*. This indicates that C is an ungrounded denoting phrase. The repetition of the primary representation shows that C cannot be given denotation conditions by the c_C-schema - and so we can also say that C is a *singularity* of 'denotes$_{cC}$'.

We can now see why it is that the primary representation of C plays a privileged role in the determination of C as pathological. If we evaluate C by the c_C-schema - in accordance with C's primary representation - then C and the members of its determination set are evaluated by the same schema, since the second and third elements in C's primary representation are identical. And, since C is a member of its own determination set, the primary representation repeats endlessly on C's primary tree. The primary representation of C, and the primary tree generated from it, bring out the way in which C is caught in a circle of evaluations. If we want to capture the semantical status of C, in particular its pathology, we must work with its primary representation.

The same holds of C*. If we want to capture *its* semantical status, we must begin with its primary representation. The primary representation of C* is $\langle \text{type(C)},c_C,r_C \rangle$. The members of C*'s determination set are A, B and C. So the primary tree for C* is:

$\langle \text{type(C)},c_C,r_C \rangle$

/ | \

type(A) type(B) $\langle \text{type(C)},c_C,c_C \rangle$

/ | \

type(A) type(B) $\langle \text{type(C)},c_C,c_C \rangle$

/ | .

type(A) type(B) .

.

The primary representation of C repeats on the infinite branch, *but the primary representation of C* does not*. The primary representation of C* stands above the circle in which C is caught.

This indicates that C* is not pathological, and is not a singularity of 'denotes$_{rC}$'. C's pathology is not the end of the matter - we can reason past pathology. When we determine a value for C*, we will need to determine the denotation$_{cC}$ of the members of its determination set. Since C does not denote$_{cC}$ -- as the tree indicates -- we determine a value for C* in terms of A and B only.

We can construct the primary trees for P and P* in parallel fashion. The primary tree for P is:

$\langle$type(P),c_P,$c_P\rangle$

/ \

type(M) $\langle$type(P),c_P,$c_P\rangle$

/ .

type(M) .

.

The primary representation of P repeats on the infinite branch, indicating that P is a pathological predicate, and a singularity of 'extension$_{cP}$'. The primary tree for P* is:

$\langle$type(P),c_P,$r_P\rangle$

/ \

type(M) $\langle$type(P),c_P,$c_P\rangle$

/ \

type(M) $\langle$type(P),c_P,$c_P\rangle$

/ .

type(M) .

.

P* - like C* - is *not* pathological, since the primary representation of P* does not repeat on the infinite branch. The primary representation of P does repeat, indicating that P is pathological, and a singularity of 'extension$_{cP}$'. So we determine an extension$_{rP}$ for P* in via M only.

Similarly with L and L*. L's determination set has just one member, L itself. The primary tree of L is composed of a single, infinite branch

$$
\begin{array}{c}
\langle \text{type}(L), c_L, c_L \rangle \\
| \\
\langle \text{type}(L), c_L, c_L \rangle \\
| \\
\langle \text{type}(L), c_L, c_L \rangle \\
. \\
. \\
.
\end{array}
$$

Since the primary representation of L repeats on this branch, L is pathological. And L is identified as a singularity of 'true' as it occurs in L – it cannot be evaluated by the c_L-schema.

L*'s context is reflective with respect to L, and the primary tree for L* is:

$$
\begin{array}{c}
\langle \text{type}(L), c_L, r_L \rangle \\
| \\
\langle \text{type}(L), c_L, c_L \rangle \\
| \\
\langle \text{type}(L), c_L, c_L \rangle \\
. \\
. \\
.
\end{array}
$$

The primary representation $<type(L),c_L,r_C>$ of L* does not repeat on this infinite branch: this indicates that L* is not pathological, and that L* can be assessed by the r_L-schema. Since the primary representation of L repeats, the tree indicates that L is a singularity of ‘$true_{cL}$’, and so isn't in the extension of ‘$true_{cL}$’. So we can declare L* $true_{rL}$, because it says that L is not in the extension of ‘$true_{cL}$’.

In general, if an expression token σ is pathological, then its primary tree, generated from its primary representation, will display its pathology. Pathology arises because the token is looped or in some way tangled with certain members of its determination set, certain members of the determination sets of those members, and so on. By starting with σ's primary representation, we ensure that the context of σ is not inappropriately treated as reflective with respect to a member of its determination set. Such an inappropriate treatment may cover up σ's pathology. For example, consider the secondary representation $<type(C),c_C,r_C>$ of C - this is the primary representation of the non-pathological token C*, and if we represent C this way, we will not reveal C's pathology. And so when it comes to determining the semantic status of σ, it is the primary representation of σ, and not some secondary representation, that has a privileged role to play.

That is not to say that secondary representations have no role to play. Recall *rehabilitation*. The fourth and final stage of the rehabilitation discourse associated with C, for example, runs as follows (with the contextual subscript attached):

(R) So the phrase C - made up of the words *the sum of the numbers denoted*$_{cC}$ *by expressions on the board in room 213* – denotes$_{cR}$ π+6.

The evaluation of C in the final sentence is captured by a secondary representation of C, namely $<\text{type}(C),c_C,c_R>$. Since the c_R-schema is a reflective r_C-schema, we can also write this secondary representation as $<\text{type}(C),c_C,c_R>$. This secondary representation generates the following *secondary tree* for C:

$<\text{type}(C),c_C,r_C>$

/ | \

type(A) type(B) $<\text{type}(C),c_C,c_C>$

/ | \

type(A) type(B) $<\text{type}(C),c_C,c_C>$

/ | .

type(A) type(B) .

.

This tree captures the reasoning of the final stage of rehabilitation. In order to reflectively determine a denotation for C, we must determine the denotations$_{cC}$ of the members of its determination set. Working along the second tier of the tree, the denotations of A and B are straightforwardly determined. But the primary representation of C at the second tier repeats on an infinite branch. So C has no denotation$_{cC}$, and so we reflectively establish a denotation for C via A and B alone. The secondary tree for C is identical to the primary tree for C* -- this is what we would expect, since this secondary representation of C is identical to the primary representation of C*. This is always so for a pathological expression ρ and a repetition ρ* -- the primary tree for ρ* is identical to a secondary tree for ρ, and ρ's secondary tree indicates that ρ can be assessed by the r_ρ-schema.

Suppose now we assess C by the schema associated with some neutral context n, quite unrelated to the contexts of C and C*. Here the secondary representation of C will be the triple $<\text{type}(C),c_C,n>$. The *secondary tree* for C is:

$\langle \mathrm{type}(C), c_C, n \rangle$

/ | \

$\mathrm{type}(A) \quad \mathrm{type}(B) \quad \langle \mathrm{type}(C), c_C, c_C \rangle$

/ | \

$\mathrm{type}(A) \quad \mathrm{type}(B) \quad \langle \mathrm{type}(C), c_C, c_C \rangle$

/ | .

$\mathrm{type}(A) \quad \mathrm{type}(B)$.

.

The secondary representation of C does not repeat on the infinite branch. This indicates that C does denote$_n$: a denotation for C can be determined by the n-schema. What C denotes$_n$ will depend on what the members of its determination set denote$_{cC}$. And since C is a singularity of denotes$_{cC}$, what C denotes$_n$ will depend only on what A and B denote$_{cC}$. C is *not* a singularity of 'denotes$_n$', in accordance with Minimality. Here the secondary tree captures the idea that the neutral context n is *non-explicitly reflective* with respect to C. Similarly with P and L.

Expressions may be pathological because they are looped with other expressions. For example, consider the loop in which Fran and Grace are caught, introduced in the previous chapter. Fran says:

(F) the sum of 2, 3, and the number denoted by Grace's current utterance,

and Grace says:

(G) the sum of 2, 3 and the number denoted by Fran's current utterance.

The primary tree for F is given by:

$\langle \text{type}(F), c_F, c_F \rangle$

/ | \

$\langle \text{type}(2) \rangle$ $\langle \text{type}(3) \rangle$ $\langle \text{type}(G), c_G, c_F \rangle$

/ | \

$\langle \text{type}(2) \rangle$ $\langle \text{type}(3) \rangle$ $\langle \text{type}(F), c_F, c_G \rangle$

/ | \

$\langle \text{type}(2) \rangle$ $\langle \text{type}(3) \rangle$ $\langle \text{type}(G), c_G, c_F \rangle$

/ | \

$\langle \text{type}(2) \rangle$ $\langle \text{type}(3) \rangle$ $\langle \text{type}(F), c_F, c_G \rangle$

/ | .

$\text{type}(2) \rangle$ $\langle \text{type}(3) \rangle$.

.

The secondary representations $\langle \text{type}(F), c_F, c_G \rangle$ and $\langle \text{type}(G), c_G, c_F \rangle$ repeat on the infinite branch. This indicates that F cannot be assessed by the c_G-schema, and G cannot be assessed by the c_F-schema. But now, by Symmetry, F and G are to be treated alike, so that if one cannot be assessed by a given schema, neither can the other. So, since G cannot be assessed the c_F-schema, neither can F. And since F cannot be assessed by the c_G-schema, neither can G. So F and G are both singularities of ‘denotes_{cF}’ and of ‘denotes_{cG}’.

The simplest kind of loop, exhibited by C, is associated with a primary tree in which the top node repeats on an infinite branch. Here an expression is directly looped with itself. Wider loops, like the Fran-Grace loop, are composed of a network of distinct expressions, σ, ρ, τ … , each expression of which makes reference to the next, cycling back to σ. Consider the primary tree for σ: there will be an infinite branch whose top node is the primary representation of σ, and whose subsequent nodes will be secondary representations of ρ, τ, …σ. The secondary representations will repeat on this infinite branch. Even though σ’s primary representation does not repeat on this branch, it is a consequence of Symmetry that the branch indicates σ's pathology. Since ρ is a determinant of σ, the secondary representation of ρ will be of the form $<\text{type}(\rho),c_\rho,c_\sigma>$. The representation $<\text{type}(\rho),c_\rho,c_\sigma>$ repeats, showing that ρ cannot be assessed by the c_σ-schema. By Symmetry, since ρ cannot be assessed by the c_σ-schema, neither can σ. So σ is pathological – it cannot be evaluated by its associated schema.

In general, we can say that an expression σ *loops* if σ's primary tree contains an infinite branch on which either the primary representation of σ or a secondary representation of σ repeats. (It's easy to show that if a secondary representation of σ repeats on an infinite branch, then all secondary representations on that branch repeat.)

Besides loops, there are chains, as we saw in the previous chapter. For example, consider again the sequence of daily, single utterances at the Great Rock:

(1)
$$m = \begin{cases} \text{the number denoted by tomorrow's utterance here} \\ 0\text{, if there is no such number.} \end{cases}$$

(2)
$$m = \begin{cases} \text{the number denoted by tomorrow's utterance here} \\ 0\text{, if there is no such number.} \end{cases}$$

Each day a token of the same type is produced, and so on, forever. Each of the expressions (1), (2), (3), ... (n), ... heads an infinite chain. The primary tree for (1) is this:

$$
\begin{array}{c}
\langle \text{type}(1), c_1, c_1 \rangle \\
| \\
\langle \text{type}(2), c_2, c_1 \rangle \\
| \\
\langle \text{type}(3), c_3, c_2 \rangle \\
. \\
. \\
.
\end{array}
$$

The tree is composed of a single branch, and the branch is infinite, indicating pathology. No representation repeats on this branch, and that is the distinctive feature of a chain. The branch indicates that (2) cannot be evaluated by the c_1-schema, (3) cannot be evaluated by the c_2-schema, and so on. By Symmetry, since (2) cannot be evaluated by the c_1-schema, neither can all the other utterances in the network; and since (3) cannot be evaluated by the c_2-schema, neither can all the other utterances in the network. So (1), (2), (3), … are all singularities of the occurrences of 'denotes' in (1), (2), (3), … .

We can now give, in a rough and ready way, a more general characterization of the notions of *semantic pathology* and *singularity*. (Here we anticipate the formal theory to be developed in Chapter 6.) Let σ be a phrase token containing an occurrence *t* of 'denotes', 'extension' or 'true'. σ's primary representation is $\langle \text{type}(\sigma), c_\sigma, c_\sigma \rangle$. σ is *pathological* if σ's primary tree contains an infinite branch such that

(a) the primary representation of σ or a secondary representation of σ repeats on the branch, or

(b) no representation on the branch repeats.

Further, let <type(ρ),c_ρ,δ>, representing the expression token ρ, be any node on an infinite branch of σ's primary tree. Then ρ is a *singularity* of 't_δ' - the infinite branch indicates that a denotation for ρ cannot be determined by the δ-schema. (In particular, σ itself is a singularity of '$t_{c\sigma}$ '.)

Now suppose that we are trying to determine a value for σ. The members of σ's determination set are assessed by the c_σ-schema. If σ is pathological, some of these members will be singularities of '$t_{c\sigma}$'. The exclusion of these members from the extension of '$t_{c\sigma}$' will be part of the procedure that determines a value for σ (if σ has a value). Roughly speaking, we can think of σ's primary tree as displaying the information we need to determine reflectively a value for σ. For example, consider the primary tree for C. At its second tier are the representations of the members of its determination set, and we must determine what these members denote$_{cC}$ if we are going to determine a value for C. It is a straightforward matter that A denotes$_{cC}$ π, and B denotes$_{cC}$ 6. But the infinite branch indicates that C - a member of C's determination set - does not denote$_{cC}$ at all. And so we determine a denotation for C by determining the denotations$_{cC}$ of A and B only. This procedure captures the way in which we establish a value for C via a reflective r_C-schema.

With the notion of a singularity on board, we can state general principles of denotation, extension and truth that are *minimally restricted*. Given any context α, consider a token of the type 'denotes' or 'extension' or 'true' that is represented by 'denotes$_\alpha$' or 'extension$_\alpha$' or 'true$_\alpha$'. For the case of denotation, the principle is this:

(i) If e is not a singularity of 'denotes$_\alpha$', then e denotes$_\alpha$ k iff d=k,

and (ii) if e is a singularity of 'denotes$_\alpha$', then e does not denote$_\alpha$, where d is replaced by a denoting expression, and e is replaced by a name of that expression.[3] In the case of extension, the principle is:

(i) If φ is not a singularity of 'extension$_\alpha$', then

for all x, x is in the extension$_\alpha$ of φ iff x is Φ,

and (ii) if φ is a singularity of 'extension$_\alpha$', then φ has no extension$_\alpha$,

where x ranges over objects, Φ is replaced by a predicate, and φ is replaced by a name of that predicate. For truth, the principle is:

(i) if s is not a singularity of 'true$_\alpha$', then s is true$_\alpha$ iff S,

(ii) if s is a singularity of 'true$_\alpha$', then s is not true$_\alpha$ (or false$_\alpha$),

where S is replaced by a sentence, and s is replaced by a name of that sentence. We'll discuss these principles further in Chapter 9.

4.2 Singularities and Semantic Universality

Natural languages are remarkably flexible and open-ended. If there is something that can be said, it might seem that a natural language like English has at least the potential to say it. Natural languages evolve; they always admit of extension, of increased expressive power. Tarski speaks of the "all-comprehensive, universal character" of natural language, and continues:

> "The common language is universal and is intended to be so. It is supposed to provide adequate facilities for expressing everything that can be expressed at all, in any language whatsoever; it is continually expanding to satisfy this requirement."[4]

In the same vein, Tarski writes:

> "A characteristic feature of colloquial language (in contrast to various scientific languages) is its universality. It would not be in harmony with the spirit of this language if in some other language a word occurred which could not be translated into it; it could be claimed that 'if we can speak meaningfully about anything at all, we can also speak about it in colloquial language'.[5]

We should avoid misunderstandings about Tarski's notion of universality. To claim that natural languages are universal in Tarski's sense is not to claim that all concepts are expressible in natural language. This latter claim would be highly controversial. Consider, for example, the sets in the ZF hierarchy: for each such set, it is arguable that there is a unique concept of being identical to that set. Given certain assumptions about natural languages (in particular, about upper limits on the size of vocabularies, and on the length of linguistic expressions), these concepts would outrun the expressive capacity of any natural language. But Tarski does not make the claim that natural languages can express all concepts. Rather, Tarski is claiming that *if* a concept is expressible in *some* language, then it is expressible in any natural language. This claim is perfectly compatible with the existence of concepts that are inexpressible (in every language).

In particular, Tarski says, natural languages are *semantically universal*. According to Tarski's characterization, a semantically universal language contains names for its own expressions, contains its own semantical predicates that apply to expressions of the language, and has the resources for describing the proper use of these expressions.[6] In short, a semantically universal language can say everything there is to say about its own semantics.

We would need further argument to make out the claim that English is universal in Tarski's sense. For example, it is possible to take issue with Tarski's claim that any word of any natural language may be translated into any other natural language.[7] Good translations, even adequate ones, are often hard to come by.[8] However, a natural language like English *does* appear to be semantically universal. Consider, for example, the predicate 'denotes'. This is an English predicate that applies to denoting expressions of English (strictly speaking, to all ordered pairs of English expressions and their referents). And the application of 'denotes' appears to be *global*, in the sense that it applies to every denoting expression of English -- and of other languages too. And we seem to be able to describe the proper use of this term in English. It is the same for ‘extension’ and ‘true’. So it seems to me that we should respect as far as possible Tarski's intuition that natural languages like English are semantically universal.

I think that the singularity proposal goes a long way to accommodate this intuition. An occurrence of ‘denotes’ or ‘extension’ or ‘true’ is as close to universal as it can be without contradiction – with the exception of its singularities, it applies to all denoting expressions, or all extensions, or all truths. Moreover, according to the singularity proposal, even expressions that are singularities relative to a given context fall into the extensions of occurrences of 'denotes', ‘extension’ or ‘true’ in other contexts (such as an appropriate reflective context, or an appropriate neutral context). So the application of any occurrence of a semantical predicate is almost global, and those expressions that prevent its application from being fully global are captured by other uses of the predicate. We will return to the topic of semantic universality in a broader setting in Chapters 8 and 9, when we consider revenge paradoxes.

4.3 Comparisons

Attention has been lavished on the simple Liar paradox, and I take up some of this discussion in the next chapter. Our simple Russell paradox and the simple paradox of denotation have received far less attention. As far as I know, the simple Russell has never been presented or discussed previously. There has been some recent discussion of our simple paradox of denotation, and I think it's instructive to compare these alternative proposals with the singularity proposal.

Consider the singularity treatment of the simple paradox of denotation generated by C. I claim for it a number of positive features. It requires no revision to classical logic. It stays true to the way in which we, as ordinary speakers, reason with 'denotes' when exposed to the paradoxes. In particular, it respects the intuitiveness of the reasoning associated with *repetition* and *rehabilitation* -- rather than artificially blocking the reasoning, it affirms its soundness. The context-sensitivity attributed to 'denotes' in the accounts of *repetition* and *rehabilitation* is based on a well-entrenched view of context, one which accords well with our ordinary reasoning with 'denotes'. And the changes in extension with contextual shifts are kept to a minimum, so that each occurrence of 'denotes' is as global in its application as it can be. In this way, we respect the intuition that natural languages are universal – and, in Gödel's words, our logical intuitions "remain correct up to certain minor corrections".

I've argued that the singularity account compares favorably with a hierarchical account of denotation: the hierarchical account is too regimented and artificial, there is no evident way of assigning levels, and massive restrictions are imposed on ordinary uses of 'denotes'. I think that the singularity account also compares favorably with two other treatments of our simple paradox of denotation, one due to Hartry Field, and one due to Kevin Scharp.[9]

Field's treatment pushes the problem over to set theory. The definite description in C is to be treated as 'the sum of the members of the set of numbers denoted by expressions on the board'. And then the diagnosis of the paradox is this: "it is illegitimate to assert the existence of sets defined by conditions for which we have no license to assume excluded middle".[10] It is neither legitimate to assert the existence of the set in question, nor legitimate to assert the *non*-existence of the set in question. According to Field, "it's illegitimate to assert one way or the other whether the description denotes and again paradox is avoided".[11] This approach seems to me to have a number of costs. It rejects classical logic; it shifts the focus away from denotation, focusing instead on the conditions under which a set is determined; and it provides no account of the intuitive reasoning exhibited in *repetition* and *rehabilitation*. It's also curious that Field does not treat the simple paradox of denotation in the same way as he treats König's and Berry's paradoxes.[12] As we'll see in the next chapter, the singularity approach treats all of the paradoxes of denotation in the same way.

According to Kevin Scharp, we should respond to our simple paradox of denotation by declaring denotation to be an *inconsistent concept*. Here is Scharp's leading example of an inconsistent concept:

> "… consider the following definition:
> (1a) 'rable' applies to x if x is a table.
> (1b) 'rable' disapplies to x if x is a red thing."[13]

The problem with *rable* is that its constitutive principles (1a) and (1b) are inconsistent (here, not in the sense of logically inconsistent, but in the sense of having false consequences, such as there being no red tables). In the case of denotation, its constitutive principle is the denotation schema from Chapter 2, which we can express in Scharp's terms this way:

> 'b' denotes a iff a=b.[14]

Scharp takes the lesson of our simple paradox to be that this principle is inconsistent (here, in the sense of generating a logical inconsistency) – and so denotation is an inconsistent concept. But denotation is also a useful concept, so it should be replaced by consistent concepts that will do the useful work that denotation does.

Scharp proposes that we replace denotation with two concepts, *Ascending denotation* and *Descending denotation*, defined as follows:

(Ddenotation) If 'b' Descending denotes a, then a=b.

(Adenotation) If a=b, then 'b' Ascending denotes a.

We'll now have two versions of the simple paradox of denotation, one where 'denotes' in C is replaced by 'Ascending denotes':

(C_{Ad}) The sum of the numbers Ascending denoted by expressions on the board,

and one where 'denotes' in C is replaced by 'Descending denotes':

(C_{Dd}) The sum of the numbers Descending denoted by expressions on the board.

But neither of these versions generates contradictions.

First consider attempts to derive a contradiction in the case of the expression C_{Ad}. Scharp shows that one attempt would require the inference from

(i) 'The sum of the numbers Ascending denoted by expressions on the board' Ascending denotes π+6

to

(ii) The sum of the numbers Ascending denoted by expressions on the board is π+6.

But this inference fails – Ascending denotation does not work in this direction. Another attempt requires the inference from

(ii) The sum of the numbers Ascending denoted by expressions on the board is π+6

to

(iii) 'The sum of the numbers Ascending denoted by expressions on the board is π+6'
Descending denotes π+6.

But this inference also fails – Descending denotation does not work in this direction. Symmetrical consideration apply to attempts to derive a contradiction in the case of C_{Dd}.[15]

According to Scharp, Ascending denotation and Descending denotation are concepts that together can do the useful work that denotation does, but do it consistently. Clearly these are technical notions. The constitutive principle for Ascending denotation does not license the move from

'b' Ascending denotes a

to

b=a,

and this is at odds with our ordinary intuitions about denotation. Similarly with the principle for Descending denotation, which does not license the move from

b=a

to

'b' Descending denotes a.

Scharp's account is clearly at odds with the singularity approach. From the point of view of the singularity theory, we have no motivation to declare denotation an inconsistent concept, or to replace it by other concepts. Scharp's account provides no account of *repetition* or *rehabilitation* – discourses associated with our simple paradox of denotation are dismissed as the flawed product of reasoning with an inconsistent concept. We should take the reasoning no more seriously than we should take reasoning with *rable* that leads to the conclusion that there are no red tables. In contrast, the singularity approach treats the *repetition* and *rehabilitation* discourses as exhibiting intuitive, sound reasoning that needs explanation, an explanation that should revise as little as possible our ordinary intuitions about denotation. According to the singularity account, there is no need to replace denotation -- to use Gödel's words again, we have "an essentially correct, only somewhat 'blurred', picture of the real state of affairs." We can maintain consistency without a conceptual upheaval – in particular, without the introduction of artificial notions such as Ascending denotation and Descending denotation, which are far removed from the conceptual repertoire of the ordinary speaker.

The point extends to extension and truth. In general, the singularity account takes *repetition* and *rehabilitation* as intuitive, sound classical reasoning that needs to be taken seriously and explained in a way that preserves our ordinary intuitions as far as possible. So the singularity theory is also at odds with Scharp's account of truth, according to which our inconsistent concept of truth is to be replaced by the artificial notions of Ascending truth and Descending truth, just as denotation is replaced by Ascending denotation and Descending denotation. And the singularity account is at odds with inconsistency views of truth generally – for example, the views of Patterson,[16] Ludwig,[17] and Eklund.[18] In contrast to these views, the singularity theory aims for consistency while still preserving *our* notion of truth, the one with which we reason.

Notes to Chapter 4

1. My thanks to Gil Sagi for emphasizing the point that there are variants of repetitions – see Sagi (ms).

2. For simplicity, we will in general ignore all context-sensitive expressions other than 'denotes'. So we will always represent a denoting phrase via its type unless it contains an occurrence of 'denotes'.

3. As we noted in Chapter 2 (note 20), there must be restrictions on the c_C-schema and the c_E-schema. Suppose we admitted an unrestricted c_C-schema:

‘p’ denotes_{cC} n iff p=n

and an unrestricted c_E-schema:

‘p’ denotes_{cE} n iff p=n,

where ‘p’ is a denoting expression token. Then by the logic of ‘iff’, the two distinct contexts collapse:

‘p’ denotes_{cC} n iff ‘p’ denotes_{cE} n.

But with the notion of a singularity on board, we can state an appropriately restricted principle of denotation.

Jc Beall has objected to these restricted denotation principles, as follows (Beall 2003a, pp.260-61). Consider the claim ‘C does not denote_{cC}’, made in the course of the strengthened reasoning. Beall asks what the import or content of this claim is, and finds that no clue is given by the principle:

If e is not a singularity of ‘denotes$_{cC}$’, then e denotes$_{cC}$ k iff d=k.

Beall argues that the meaning of ‘denotes$_{cC}$’ is given only when we’ve detached the (biconditional) consequent – and this we cannot do in the case of C, since C is a singularity of ‘denotes$_{cC}$’.

But this objection gets things the wrong way round. Let us accept that the biconditional consequent gives the denotation$_{cC}$ conditions for denoting expressions – in that sense, at least, we may regard it as meaning-giving. Now in the course of the strengthened reasoning we discover that C cannot be given denotation-conditions. Far from requiring the biconditional schema to give meaning to the claim that C does not denote$_{cC}$, we find that the schema cannot supply denotation-conditions for C. C is not in the extension of ‘denotes$_{cC}$’ (more precisely, C is not the first member of any of the ordered pairs that comprise the extension of ‘denotes$_{cC}$’). And this is all there is to the content of the claim that C does not denote$_{cC}$ – not only we do not need the biconditional schema to supply this content, the schema *could not* supply it.

Contrast C with, say, the phrase ‘the square of 1’. Consider the claim ‘”The square of 1” does not denote$_{cC}$ 2’. We can give the truth conditions of this claim via the biconditional schema. From the instance

‘The square of 1’ denotes$_{cC}$ 2 iff the square of 1 is identical to 2,

we can infer

‘The square of 1’ does not denote$_{cC}$ 2 iff the square of 1 is not identical to 2.

But of course we cannot say that ‘the square of 1’ does not denote$_{cC}$, since it does (it *is* the first member of an ordered pair in the extension of ‘denotes$_{cC}$’). When we say that C does not denote$_{cC}$, we are excluding C altogether from the extension of ‘denotes$_{cC}$’, and placing it outside the reach of the biconditional schema and the denotation$_{cC}$ conditions that the schema provides.

4. Tarski 1969, p. 89.

5. Tarski 1986, p.164.

6. Tarski uses the term 'semantically universal' in Tarski 1969, p.89, and 'semantically closed' in Tarski 1944, p.20.

7. See for example Ziff 1988, p.8.

8. For a discussion of this, see Bar-On 1993.

9. In Priest 2004, Graham Priest defends a dialetheist solution to the paradoxes of denotation, noting that, for some versions of the paradox, it is not enough to simply accept the contradiction as true (see p.122). I discuss dialetheism in Chapter 8.

10. Field 2008, p.294, fn.7.

11. *ibid*.

12. According to Field, definability paradoxes like König's and Berry's show that the notion of definability in a given language does not have sharp boundaries (in common with a vague term like 'old'). In response we should restrict the law of excluded middle for the notion of definability (in a given language). Determinateness enters the picture, as it does with vagueness, and we cannot say of a Berry- or König-like phrase that it either determinately defines or determinately fails to define a number within the given language. In his resolution of König's and Berry's paradoxes, Field makes no mention of sets or the defining conditions for sets – but it's in terms of sets and their defining conditions that Field resolves our simple paradox of denotation.

12. See Scharp 2013, p.36.

13. Here and throughout I use 'denotes' where Scharp uses 'refers to' – nothing, of course, hangs on this.

15. For full details, see Scharp 2013, 8.10.

16. See Patterson 2006, 2007, 2008, 2009.

17. See Ludwig 2001.

18. See Eklund, 2002a, 2002b.