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Paradox, Repetition, Revenge

I. Repetition

My aim in this paper is to suggest an account of paradox that requires a minimum of logical revision. But first, consider a phenomenon that cuts across the semantic paradoxes – whether a paradox of definability, or a version of Russell's paradox, or the Liar. The phenomenon is this: given a paradoxical expression, we can go on to produce an unproblematic expression composed of the very same words.

For example, suppose I write on the board these expressions:

pi

six

the sum of the numbers denoted by expressions on the board.

Suppose the third expression denotes a number, say k. Then we can reach the contradictory conclusion that $k = \pi+6+k$. So the third expression fails to denote a number. Only the first two expressions denote numbers. So we may truly conclude:

the sum of the numbers denoted by expressions on the board is $\pi+6$.

And here we've repeated the defective phrase, and produced an expression that does denote a number, namely $\pi+6$.

Or suppose I write on the board these two expressions:

moon of the Earth

unit extension of a predicate on the board.

The first expression has a well-determined unit extension. Suppose the second expression has an extension, say E_2. Either E_2 belongs to itself or it doesn't. If E_2 does belong to itself, then it has two members, and so doesn't belong to itself. If it doesn't belong to itself, then it has just one member – so it is a unit extension, and does belong to itself. Either way, we get a contradiction. So the second expression is defective, and does not have a well-determined extension – in particular, it does not have a unit extension. So we may conclude:

> The extension of the first expression is the only *unit extension of a predicate on the board*.

And here we've repeated the defective expression – and this repetition has a fully determinate extension, namely the extension whose only member is the Moon.

Or suppose I write on the board the following sentence:

> The sentence on the board is not true.

This is a liar sentence, and it's semantically defective. So we can conclude:

> *The sentence on the board is not true*.

These discourses display apparently natural and intuitive reasoning, and any adequate response to semantic paradox should provide some account of them. Call this phenomenon *Repetition*. Repetition is one form of *revenge*: the identification of an expression as paradoxical seems to lead to the semantic rehabilitation of that very expression. We'll see another form of revenge in Section IV below.

II. Some treatments of Repetition

Though Repetition is common to all the semantic paradoxes, it's the case of the Liar that has received most attention. Consider Kripke's theory of truth. At the center of Kripke's theory is the minimal fixed point construction. In brief, Kripke spells out a transfinite inductive process by which every sentence that *can* receive a truth value eventually *does* receive one. Liar sentences never receive a truth value in this process – they're ungrounded. So we *can* say that Liar sentences are not true, in the sense that they never receive a value in this process. So if we take our Liar sentence on the board, this sentence will be ungrounded, and gappy, neither true nor false in the minimal fixed point. Consequently, we can say that the sentence on the board is not true. Kripke's own take is that this subsequent evaluation, that the Liar sentence is not true, is expressed in a metalanguage. So Repetition is not *genuine* repetition: the object language Liar sentence is one thing, the subsequent statement that the liar sentence is not true is another, expressed in a more technical metalanguage. Kripke says:

> "If we think of the minimal fixed point, say under the Kleene valuation, as giving a model of natural language, then the sense in which we can say, in natural language, that a Liar sentence is not true must be thought of as associated with some later stage in the development of natural language, one in which speakers reflect on the generation process leading to the minimal fixed point."[1]

According to Kripke's view the 'repetition' and the original Liar sentence (L) are sharply distinguished – the repetition is expressed by speakers who speak a metalanguage containing technical notions such as *groundedness.*

According to Field,[2] we do want to accommodate the evaluation of the Liar sentence as not true. But what is needed, he thinks, is a stronger notion of truth, so that when we say that Liar sentences are not true, we're saying that Liar sentences are not *strongly true*. In Field's theory, *strong truth* is to be understood in terms of a *determinately* operator applied to the notion of truth - so when we say that the Liar sentence is not true, we're saying that it's not *determinately true*. And we can express the *defectiveness* of the Liar sentence by saying that neither it nor its negation is determinately true. The notion of determinate truth in turn gives rise to a new liar – a sentence that says of itself that it's not determinately true. We cannot say that this isn't determinately true, on pain of contradiction. But we can say that it's not determinately determinately true. Iterating, we obtain a transfinite hierarchy of increasingly strong notions of truth, and corresponding notions of defectiveness.

Like Kripke's theory, Field's theory says that Repetition is not genuine repetition. The Liar sentence says of itself that it's not true; our subsequent evaluation of the Liar says that it's not true in a stronger sense – that it's not determinately true. The two sentences involve different notions of truth. Unlike Kripke's theory, Field's way of treating Repetition does not involve any ascent to a metalanguage -- the determinately operator is definable within the object language. But it does involve a hierarchy of stronger and stronger notions of truth.[3]

I think that these accounts make heavy weather of Repetition. There is a simpler way. Kripke's and Field's accounts suggest a difference between the occurrence of 'true' in the liar sentence and the subsequent occurrence in the repetition. So let's fix the use of 'true' in the liar sentence as 'true_L', where (L) is the Liar sentence in which this use of 'true' occurs. Any use of 'true', whether or not it occurs in (L), may be represented as 'true_L' as long as it is coextensive with the use of 'true' in L. The Liar sentence is represented as:

(L) The sentence on the board is not true_L.

If we suppose (L) is $\text{true}_{L,}$ we reach a contradiction. And if we suppose (L) is not $\text{true}_{L,}$ we reach a contradiction again. So we conclude that (L) is defective – it cannot be assessed by the truth_L-schema. So we *truly* conclude:

(R) The sentence on the board is not true_L.

For if (L) was true_L, it would be assessable by the truth_L-schema, and contradiction would result. Notice that, in contrast, it does not follow from (L)'s being not true_{cL} that it is assessable by the truth_L-schema. Our conclusion (R) that (L) is not true_L does not lead back to contradiction. For that we would need the truth_L-schema -- but we infer (R) just because we recognize that the truth_L-schema is no longer available as an assessing schema for (L).[4]

(R) is a genuine repetition – both sentences (L) and (R) are comprised of the same words with the same meanings and extensions. To capture Repetition, there is no need to appeal to a shift to a metalanguage, or to split truth into different notions, or to deny that we can say that (L) is not true. We have just a single truth predicate, the one we started with, with no change of extension. We represent the intuitive move from *defective* to *not true* in an appropriately simple way. It's the same story for denotation and extension. What remains to be explained is the difference in status between (L) and (R), and I'll come to that in a moment.

To reinforce the difference between this alternative account and the accounts of Kripke and Field, I want to consider a certain chain of reasoning. There are two things we'll need to carry out this reasoning. First, the capacity to produce 'anaphoric' liars. Given the sentence

'2+2=5' is not true

we're able to form this liar sentence

'2+2=5' is not true and neither is this sentence.

Second, the readiness to infer that a semantically defective sentence isn't true. Once an ordinary speaker becomes familiar with the Liar, these are modest capacities.

Now suppose we reason in the following way. We carry out the following chain of reasoning:

Since (L) is a liar sentence, (L) is defective. So

(1) (L) is not true.

Now we form a new liar sentence, building on (1):

(1*) (L) is not true and neither is (1*).

Since (1*) is a liar sentence, (1*) is defective. So

(2) (1*) is not true.

Now form a new liar sentence, building on (2):

(2*) (1*) is not true and neither is (2*).

Since (2*) is a liar sentence, (2*) is defective. So

(3) (2*) is not true.

And so on, through a series of liar sentences (3*), (4*), … .

On the present simple account that I'm suggesting, 'true' is represented by 'true$_L$' throughout. Saying that (L) is defective is to say that it does not have truth$_L$ conditions, that it cannot be assessed by the truth$_L$-schema. From this, (1) follows:

(1) (L) is not true$_L$.

We now form anaphorically the liar sentence (1*):

(1*) (1) is not true$_L$ and neither is (1*).

(1*) in turn cannot be assessed by the truth$_L$-schema, and so is not true$_L$. This is (2). And so on. We use a single truth predicate that maintains the same extension throughout. And we employ a single notion of defectiveness – the failure to be assessable by the truth$_L$-schema. This captures the fact that as we reason along this chain, we're doing the same thing over and over again.

How do the other accounts represent this reasoning? According to Kripke's account, (1) belongs to a different language from the Liar sentence (L) – in producing (1), we are speaking in a technical metalanguage that indicates reflection on the minimal fixed point. And the subsequent steps will presumably carry us through a hierarchy of further technical metalanguages. All this seems at odds with the availability of the reasoning to the ordinary speaker.

According to Field, (1) is to be interpreted as:

(1) (L) is not determinately true$_L$.

(1) involves the use of a stronger notion of truth. Now (1*) is:

(1*) (L) is not determinately true$_L$ and neither is (1*).

So (1*) is a determinately-true liar sentence. We cannot say that (1*) is not determinately true.

The sense in which we can say that it is not determinately true is in terms of a still stronger notion of truth. So (2) is interpreted as:

(2) (1*) is not determinately determinately true$_\alpha$.

In turn, (3) is interpreted as:

(3) (2*) is not determinately determinately determinately true$_\alpha$.

And so on. We're interpreted as employing a sequence of stronger and stronger notions of truth and defectiveness that are further and further removed from ordinary language. And Field does take these notions to be outside ordinary talk. "The fact is", he writes, "that people rarely iterate determinately operators very far" (p.351). Yet here we are being interpreted as iterating the determinately operator repeatedly – and this is implausible, given how little resources we need to carry out this reasoning. The reasoning repeats the same cycle of steps, and we have no reason to identify any increasing conceptual complexity.

Let's return to (L) and (R). (L) is defective, but (R) is true – and yet (R) is a genuine repetition of (L). How can we explain the difference in status? We should look for a difference in the contexts in which we produce (L) and (R). And there is a clear difference. When we produce (R), we produce it in the light of new information that has become available: *the information that (L) is defective*. It's the same in all our Repetition discourses – new information becomes salient, the information that the original expression is defective. And the repetition is produced in the light of this information. And this shift in available information produces a shift in the context. This is a familiar view of context-change. We can put it in terms of shared presuppositions (Stalnaker);[5] or the conversational score (Lewis);[6] or file-change semantics (Heim); or a dynamic theory of discourse structure (Grosz and Sidner);[7] or 'given' and 'new' information (e.g. Halliday, Chafe, Allerton).[8] But the general idea is the same: changes in the body of information presumed to be available changes the context. In all our Repetition discourses, new information becomes available – the information that the original expression is semantically defective.

Now this new information is itself *semantic* information, and it changes the standards of evaluation for (L). It changes the standards of evaluation because it is itself information *about those standards*. In recognizing that (L) is semantically defective, we recognize that (L) cannot be assessed by its associated truth$_L$-schema. We now reject the truth$_L$-schema for (L), where previously we attempted to assess (L) by that schema. And since the truth$_L$-schema does not apply to (L), we conclude that (L) is not true$_L$ (for if (L) were true$_L$, the truth$_L$-schema would apply, and contradiction would result). This conclusion – that (L) is not true$_L$ -- is (R), and this evaluation of (L) is no longer to be assessed by the truth$_L$-schema. New semantic standards are in play.

We can say that (R)'s context, unlike (L)'s context, is *explicitly reflective with respect to (L)*.[9] In general, we can identify a contextual parameter here. Given a context *c* and an expression *E*, we can ask whether or not in context *c* it is part of the information presumed to be available (or part of the common ground or conversational score) that *E* is defective – we can ask about the *reflective status of the context* ***c*** *relative to expression E*.

Whether or not a context is explicitly reflective affects the standards of evaluation in that context. (L)'s context of utterance is not reflective. Initially we assess (L) by the truth$_L$ schema. That's because (L) says that a certain sentence in true$_L$ – and so to assess (L), we have to figure out whether or not the sentence (L) is talking about is true_L or not. Of course, the sentence (L) is talking about is (L) itself. So we assess (L) by the truth_L-schema.

But now (R)'s context of utterance is reflective with respect to (L). In this reflective context, we assess (L) in the light of it defectiveness, and we can truly say that (L) is not true_L, (since it is defective and not assessable by the truth_L-schema). (R) is true in this reflective context. But (R) isn't true_L, any more that (L) is. The truth_L-schema no longer provides the standard of evaluation in (R)'s context. With the contextual change in reflective status comes a shift in the standards of evaluation. Call the new truth schema the truth_{rL}-schema, a standard of evaluation that incorporates the fact that (L) is defective. Here's the instance for (R):

$$\text{(R) is true}_{rL} \text{ iff (L) is not true}_L.$$

The right hand side holds – (L) is not true_L – and (R) is true_{rL}. When we evaluate (R) as true, we are using a reflective truth schema, a new standard of evaluation, contextually introduced by a shift in reflective status (with respect to (L)).

To sum up: I think a proper treatment of Repetition should identify a repetition as a *genuine* repetition – for example, (R) is a genuine repetition of (L). There is no *intrinsic* difference between them. The difference between (R) and (L) is instead a matter of the schemas by which they're assessed. In the Repetition discourse, (L) is assessed by the unreflective truth_L-schema. (R) is assessed by the reflective truth_{rL}-schema. The shift in the evaluating schemas is prompted by the contextual shift from an unreflective to a reflective schema.

So truth – and analogously, denotation and extension – are context-sensitive. There's a broad analogy with predicates such as 'hexagonal' or 'flat' or 'tall'. If you've just said that Italy is boot-shaped, I can say that France is hexagonal and get away with it. But if we're concerned with detailed mapping of the coastline, I can't. The extension of 'hexagonal' shifts with the standards that operate in the given context. Similarly with 'true' (and 'denotes' and 'extension'). The standards by which we assess (L) and (R) are different, because of a contextual shift in reflective status.

(L) itself can also be assessed by the r_L-schema. We can *reflectively* evaluate as true_{rL} – this fits the intuition that (L) is true, because it's defective and so not true, and that's what it says. (L) can be *rehabilitated*: it is not true_L, but it is true_{rL}.

We can give the same account of Repetition in the case of denotation and extension. In these cases too, the repetition is an exact repetition of the original defective expression – but the difference between them lies in the schema by which they're assessed. Consider, for example, the case of denotation. Suppose the third expression on the board – call it (C) -- is represented as 'the sum of the numbers denoted_C by expressions on the board'. Then so too is its repetition, call it (C*). In the course of the reasoning, (C) is assessed by the denotation_C-schema, and found to be defective. We move to a context reflective with respect to (C), and produce its repetition (C*). In this subsequent context, (C*) is assessed by a reflective denotation schema, call it the denotation_{rC}-schema:

(C*) denotes_{rC} $\pi+6$ iff the sum of the numbers denoted_C by expressions on the board is $\pi+6$.

The right-hand-side is true (only the first two expressions denote_C numbers), and so (C*) denotes denotes_{rC} $\pi+6$. Again, there is no intrinsic difference between (C) and (C*); both fail to denote_β

and both denote$_{rC}$. The sole difference is that in the course of the reasoning they are assessed by different schemas – (C) by an unreflective schema, and (C*) by a reflective schema.

It's not initially obvious that 'true' – or 'denotes' or 'extension' -- is context-sensitive. But that's not because *explicit reflective status* is an obscure feature of a context. It's straightforward to say, for any context, whether or not that context is reflective with respect to a given defective expression. That is just a matter of whether or not it is part of the common ground that the expression is defective. What makes the context-sensitivity unobvious is that discourses that exhibit the context-sensitivity of our semantic predicates are not everyday discourses. Most speakers of English do not engage with the semantic paradoxes. But once we do, we need only an ordinary grasp of truth (or denotation or extension) to carry out the reasoning associated with Repetition (and the reasoning associated with the chain of liar sentences (1*), (2*), (3*), …). And the Repetition reasoning displays a shift in reflective status, and the context-sensitivity of 'true', and 'denotation' and 'extension'.

III. A 'singularity' proposal

So far, the contextual approach to Repetition is compatible with a hierarchical account of truth (or denotation, or extension). On this kind of view, 'true$_{rL}$' has a wider extension that 'true$_{C}$'. But there's an alternative that I'd like to outline now.[10]

The alternative does not stratify truth into levels. Rather a particular use of 'true' is *minimally* restricted, applying globally except for certain 'singularities', where its application breaks down. These singularities of 'true' vary with the context. This singularity theory is in the spirit of a tantalizing remark of Gödel's. In the course of rejecting Russell's hierarchical account, Gödel says:

> "It might even turn out that it is possible to assume every concept to be significant everywhere except for certain 'singular points' or 'limiting points', so that the paradoxes would appear as something analogous to dividing by zero. Such a system would be most satisfying in the following respect: our logical intuitions would then remain correct up to certain minor corrections, i.e. they could then be considered to give an essentially correct, only somewhat 'blurred', picture of the real state of affairs."[11]

Gödel's remark suggests a way of dealing with the paradoxes in a way that minimizes logical revision. The singularity approach applies this idea to truth, denotation and extension, suggesting that a use of a semantical predicate in natural language has singularities determined by the context.

Consider again the Liar sentence (L). We can represent (L) by the ordered triple $<type(L),L,L>$,where the first member of the triple indicates the type of (L), the second indicates that the occurrence of 'true' in (L) is represented by '$true_L$', and the third indicates that the associated schema is the $truth_L$-schema, a schema unreflective with respect to (L). The repetition (R) may be represented by $<type(L),L,r_L>$. The representations of (R) and (L) share the same first two members, since type(R) = type(L), and the occurrences of 'true' in both (R) and (L) are represented by '$true_L$'. These shared members reflect the fact that (R) is a repetition of (L). But the third members differ, since the standards of evaluation in the context of (R) is different from those operating in the context of (L). The schema associated with (R) is the $truth_{rL}$-schema, which is reflective with respect to (L).

The representation of (L) by $<type(L),L,L>$ captures the schema by which (L) is assessed in the Repetition discourse. Call this the *primary representation* of (L). (L) can be assessed by other schemas, but we are primarily interested in the schema by which it is initially assessed in the course of the Repetition reasoning. Similarly, the primary representation of (R) is $<\mathrm{type}(L),L,r_L>$.

The main task of the singularity theory is to identify singularities of a given occurrence of 'true'. We understand by a *sentence* a sentence type in a context. Let the *determination set* of a given sentence S be the set of those sentences to which S makes reference. If I say "Everything Max says is true", the members of the associated determination set are the things Max says. In the case of (L), this is the unit set containing just (L). The *primary tree* for S is constructed as follows. At the top is the primary representation of S, let it be <type(S),β,γ>. At the second tier are representations of the members of S's determination set, where the third member of each of these representations is β – the members of the determination set are evaluated by the occurrence of 'true' in S. Similarly, at the third tier are representations of the members of the determination sets of sentences represented at the second tier. And so on. The primary tree for (L) looks like this:

<type(L),L,L>
|
<type(L),L,L>
|
<type(L),L,L>
.
.
.

This is a (single-branched) tree with an infinite branch, indicating semantic pathology: (L) cannot be assessed by its associated schema, the truth_L-schema. In general, if the primary representation <type(S),β,γ> of a sentence S repeats on an infinite branch of S's primary tree, then S is defective, and a *singularity* of 'true_γ'. In particular, (L) is defective, and a singularity of 'true_L'. According to the singularity theory, if S is a singularity of 'true_α', then S is excluded from the extension of ‘true$_\alpha$’. So (L) is excluded from the extension of ‘true$_L$’. So (L) isn’t true$_L$, just as (R) says.

The primary tree for (R) is:

<type(L),L,r_L>
|
<type(L),L,L>
|
<type(L),L,L>
.
.
.

The primary representation of (R) does not repeat on this infinite branch, so (R) is not defective. (R) stands above the loop in which (L) is caught. Unlike (L), (R) *can* be assessed by its associated schema, the r_L-schema: (R) *is* true$_{rL}$. This provides the account of *Repetition*.[12]

In the same way, in the cases of denotation and extension, primary trees lay out the defectiveness of the expressions on the board, and identify those expressions as singularities of the occurrences of ‘denotes’ and ‘extension’ on the board.

We can consider also more complicated cases – liar loops and chains, Curry’s paradox, and so on. In each of these cases, infinite branches on the primary trees indicate defectiveness, and singularities can be read off the tree.

Now suppose that Max produces the liar sentence (L). Suppose you say, in some neutral context,

(U) Everything Max says is true.

A leading idea of the singularity theory is that anything that can be counted as true from your neutral context should be counted as true – restrictions should only be placed on an occurrence of ‘true’ when they have to be. A use of ‘true’ is as global as it can be. Think of yourself as ominiscient, able to view all the sentences that can be counted as true from your context of utterance. In your neutral context, where you are not looped or in any way tangled with Max's utterance, you can view Max's sentence reflectively. You stand above the loop in which Max is caught, and his utterance counts as true in your context of utterance.

The situation is captured by the primary tree for your utterance. Represent the occurrence of 'true' in U by 'true_U', and let this also represent any use of 'true' that has the same extension. Consider U's primary tree. The representations at the second tier are representations of the sentences in U's determination set. The third member of each representation is U, since your assessment of Max's utterances is via the truth_U-schema. In particular, the representation of (L) is the triple <type(L),L,U>. This isn't the primary representation of (L) – it's what I'll call a *secondary* representation.[13] At the third tier, and at each tier beyond, the primary representation of (L) repeats. So there is an infinite branch of U's primary tree, involving (L):

<type(U),U,U>
|
<type(L),L,U>
|
<type(L),L,L>
|
<type(L),L,L>
.
.
.

The primary representation of (L) repeats, indicating that (L) is defective, and cannot be assessed by the L-schema. Accordingly, (L) is a singularity of 'true_L'. But the representation <type(L),L,U> does not repeat, indicating that (L) can be evaluated by the truth_U-schema; (L) is not a singularity of 'true_U'. Your evaluation of (L) is a *reflective* evaluation. Your neutral context is treated as reflective with respect to (L). This is line with the idea that an occurrence of 'true' should be as global as possible.

In the previous section, I introduced the idea of a context being *explicitly reflective* with respect to an expression. This was a matter of its being part of the common ground of a context that the expression is semantically defective. The notion of a reflective context is now expanded to include contexts that are *non-explicitly reflective* with respect to an expression. Here, the reflective status of the context is determined not by the common ground, but by the semantic networks generated by primary trees. Still, this is a natural expansion of the notion of a reflective context. Suppose you are already familiar with *Repetition* discourses. Then you can readily project from these discourses to cases where an expression's pathology is not part of the common ground. Suppose, as is likely, you produce U without knowing everything that Max says. You will nevertheless have a general recipe for determining the value of U: *determine the truth values of Max's utterances, reflectively wherever appropriate* -- where a reflective evaluation is appropriate whenever Max says something semantically defective (but not pathologically tangled with what you have said). The general recipe you draw from *Repetition* identifies *reflective status* as the contextual parameter to which 'true' is sensitive – even though you don't know the specifics of the semantic network generated by U in its context of use. The recipe captures the reasoning you *would* carry out if you knew all the facts, and with the reasoning that we *do* carry out when an expression's defectiveness is part of the common ground. So the contextual parameter *reflective status* -- whether explicit or non-explicit -- is suitably tied to how speakers reason with the truth predicate in the setting of the paradoxes.[14]

The singularity theory makes minimal assumptions about the nature of contexts. It assumes a feature of *explicit reflective status* – that is, it assumes that it's a determinate feature of a context whether or not it is explicitly reflective with respect to a given expression. That again is just a matter of whether it's explicitly part of the common ground that a given expression is defective. Beyond that, singularities of 'true' are identified not by vague or *ad hoc* pragmatic principles, but rather by semantic interrelations between sentences, as laid out by the primary trees.[15]

The overall picture is this: 'true' is a context-sensitive predicate that undergoes minimal changes in its extension with shifts of context. No occurrence of 'true' is fully global in its application – if you say: "'Snow is white' is true", it's always possible to produce an anaphoric liar by adding "but this very sentence isn't". But each occurrence of 'true' is as close to global as it can be.

IV. Revenge

Any attempt to come to grips with the semantic paradoxes faces revenge. Repetition presents one form of revenge – in the case of the Liar, the very words of the Liar sentence re-emerge as something we can truly assert. This form of revenge forces Kripke to accept ascent to a metalanguage, and, the case of Field's theory, leads to a transfinite sequence of stronger and stronger notions of truth. In contrast, Repetition is a prime motivation for contextualism about truth.

A second form of revenge generates new paradoxes generated by the very terms of the purported solution. In the case of Field's paracomplete theory, for example, there is the threat of new paradoxes that emerge from Field's determinacy hierarchy. We have a hierarchy of distinct truth concepts – determinate truth, determinate determinate truth and so on – each with its own correlative notion of defectiveness. Now it seems reasonable to suppose that we have a *general* notion of determinate truth, and a *general* notion of defectiveness – but on pain of paradox, such notions cannot be expressed. In general, revenge presents a dilemma: accept expressive incompleteness or accept inconsistency.

So we might embrace inconsistency for the sake of expressive completeness. But I think that there are revenge paradoxes even for the dialetheist. In this section I'll present a revenge paradox for the dialetheist, a paradox that suggests we have little to gain by embracing true contradictions. And then, in the final section, I'll indicate how the singularity approach responds to revenge. If this response is adequate, it suggests that we don't have to give up classical logic to deal with revenge.

In what follows, I'll consider this version of dialetheism: there are only two truth values, true and false, and different sentences relate to these values in different ways – some relate consistently and some inconsistently.[16] Now it is natural to think that a revenge liar for the dialetheist is generated by the sentence:

(X) (X) is false only.

(X) is a Liar sentence, so according to the dialetheist, it is both true and false. Since it is true, (X) is false only. So (X) is both true and false, and false only. And now we might claim that (X) cannot be both true and false, *and* false *only*. But the dialetheist has a response: by dialetheist's lights, being true and false does not preclude being false only. That (X) is false only is additional information, additional to the information that it is true and false. The dialetheist will say that we can capture the status of (X) by this semantic profile:

$$T(X) \ \& \ F(X) \ \& \ {\sim}T(X),$$

where the third conjunct simply adds more information, and doesn't 'take back' the truth of (X).

So we need to go beyond the false-only paradox. Define the *value set* of a sentence S as the set of S's values. The value set of '2+2=4' is {1}; the value set of '2+2=5' is {0}; the value set of the liar sentence (X) is {1,0}. A sentence can have only one value set. Non-paradoxical sentences that are true have the value set {1}, non-paradoxical sentences that are false have the value set {0}. And paradoxical sentences, according to the dialetheist, have the value set {1,0}. If a sentence S is paradoxical, so that its value set is {1,0}, we can *at least* say this about S:

T(A) & F(A)

But this may not be its complete semantic profile. There may be more to say about (A) -- (A) may also be, for example, untrue or false only. But (A)'s value set will still be {1,0}. For example, the value set of (X) is {1,0}, even though its semantic profile is:

T(X) & F(X) & ~T(X),

where there is a conjunct beyond T(X) and F(X). In general, no conjunct of a sentence S's semantic profile other than T(A) or F(A) can contribute to the *value set* of S – though further conjuncts may add to the *semantic profile* of S. If S is paradoxical, its value set is {1,0}; A's value set cannot be smaller or larger.

Now consider the sentence:

(Y) The value set of (Y) is {0}.

Suppose first that the value set of (Y) is {1}. So (Y) is true. By the truth-schema - which Priest endorses - it follows that the value set of (Y) is {0}. Since any sentence has just one value set, it follows that {1}={0}, so 1=0, and everything is true. This is unacceptable to the rational dialetheist. Suppose second that the value set of (Y) is {0}. Then, by the truth-schema, given what (Y) says, (Y) is true. So the value set of (Y) is either {1} or {1,0}. Either way, 1=0 again. Suppose third that (Y) has value set {1,0}. Then (Y) is true (as well as false). By the truth-schema, the value set of (Y) is {0}. So, since (Y) has only one value set, {1,0}={0}, and so again 1=0. All three cases lead to triviality. So (Y) generates a revenge liar for the dialetheist.

This revenge liar does not preclude a sentence from being false only *and* true. The semantic profile of (Y) can be given as:

$$T(Y) \ \& \ F(Y) \ \& \ {\sim}T(Y).$$

But from this we can still read off the value set for (Y) – it's given by the first two conjuncts as {1,0}.

A dialetheist might respond that just as a sentence can be false only *and* true, so a sentence can have value set {0} *and* be true. But if a sentence (A) has value set {0}, then F(A) is a conjunct of its semantic profile. And if (A) is also true, then T(A) is another conjunct of its semantic profile. That establishes that the value set of A is {1,0}. So again {1,0}={0}, and 1=0.

The dialetheist might also respond by saying that (3) has value set {0} and *doesn't* have value set {0}. But if (3) doesn't have value set {0}, then it has value set {1} or {1,0}. And since (3) also has value set {0}, the result again is 1=0.

Or the dialetheist might challenge the assumption that a sentence can only have one value set. The dialetheist might say that a sentence A can have more than one value set – in particular, sentence (Y) has value set {0} *and* value set {1,0}. But then the semantic profile of (Y) will be F(Y) & T(Y) & ~T(Y). And by the definition of a value set, and the way it can be read off the semantic profile, it follows that Y's value set is {1,0}. A sentence cannot have this semantic profile and its value set be {0}. Once the conjunct T(Y) appears in (Y)'s semantic profile, (Y)'s evaluation set has 1 as a member.

The dialetheist might challenge the very intelligibility of the notion of a value set. But if we accept that there is a complete accounting of the relations – consistent and inconsistent - that a sentence bears to the values true and false, then the notion of a value set is not only intelligible, but perfectly intuitive. The notion depends only on there being a fact of the matter, for any given sentence, about the relations it bears to the values true and false. Either T(A) appears as a conjunct of A's semantic profile or it doesn't; either F(A) appears or it doesn't. To deny the intelligibility of the notion of a value set is to deny that there is a fact of the matter whether or not T(A) and F(A) are conjuncts in the semantic profile of certain sentences. But then the semantic profile of a liar sentence would be something essentially incompleteable, or unstable, or indeterminate. And the dialetheist will reject any treatment of the Liar in these terms. Indeed, the dialetheist is committed to the value set of a liar sentence being completeably, determinately and stably {0,1}.

An alternative is to allow that the notion of a value set is intelligible, but not expressible in the dialetheist language. But then the dialetheist language is expressively incomplete. And the hope was that recognizing true contradictions would make room for expressive completeness.

V. Revenge and the singularity theory

Revenge is a challenge for the singularity theory too. Let me sketch the way in which the singularity theory responds to the challenge.[17] To fix ideas, start with a fragment of English that contains no semantic terms. We obtain the language £ by adding the English predicate 'true'.

(We can do the parallel thing for denotation and extension.) £ is the language that the singularity theory is a theory of. The key claim here is this: the language in which the singularity theory is couched – call it T – is *not* a Tarskian metalanguage for £. A full defense of this claim would require a more detailed account of the singularity theory, but perhaps I can indicate the broad outlines here.

The job of the singularity theory is to identify singularities of a given occurrence of 'true'. This is done via the primary trees. The resources needed for this are relatively meager (the theory is built out of the notions of *determination set, primary representation,* and *primary tree*.) Nowhere does the language T of the singularity theory contain a predicate coextensive with any occurrence of 'true'. The theory does not provide a 'model' of English. Rather, it provides the means for identifying singularities of a given occurrence of 'true'. T is not a metalanguage for £.

Still, it is natural to go further and provide glosses on the theory that make the context-sensitivity explicit. With the singularity treatment in mind of, say, the Liar sentence (L), we should be able to say that (L) is not true when assessed by its associated truth_L-schema, but true in when assessed by the reflective r_L-schema – or more compactly, that (L) is not true in its context of use but is true in (R)'s context of use.[18] And we should be able to say that (L) is true in *some* contexts, but not in others. Recanati points out that 'small' is an indexical – but we can make the comparison class explicit ('small for an elephant'), and then expression no longer functions as an indexical.[19] Similarly, 'true' is context-sensitive – it's sensitive to the standards of evaluation that are operative in a given context. But we can make those standards explicit: for example, 'true in the reflective context r_L'. The predicate 'true' is like 'small' or 'hexagonal' or 'flat' in that it can be used as a context-sensitive expression, or as part of a decontextualized complex term.   So ‘true’ (and ‘denotes’ and ‘extension’) are not *narrow* indexicals, if by narrow indexicals we mean indexicals that are non-bindable.[20]

So suppose we consider all the sentences of £ that are true in *some* context. Let’s introduce ‘true$_{£}$’ to abbreviate the phrase “sentence of £ that is true in some context”. We can think of this predicate as the truth predicate for £. It applies to “2+2=4” and to the Liar sentence (L), and in general to any sentence of £ that is true in some context.   Suppose we add ‘true$_{£}$’ to the language T of the theory – call this language T+.

Now there are certain things that can be said in T+ that cannot be said in £, on pain of paradox. That’s because every occurrence of ‘true’ in £ has singularities – as we saw, we can always construct anaphoric liars. And these singularities will be true in some suitably reflective context, and so they will be true$_{£}$. So ‘true$_{£}$’ will in this way apply more broadly that any given occurrence of ‘true’. So in this way T+ is expressively richer than £.

But in other ways, £ is richer than T+. T+ is a restricted theoretical language, free of context-sensitive terms, and since its sentences are not identified as singularities, every occurrence of ‘true’ will have the sentences of T+ in its scope. Since we can regard T+ as a classical formal language, it is subject to Tarski’s theorem, and a Tarskian hierarchy can be generated from it. But since none of the theoretical sentences in this hierarchy are identified as singularities of ‘true’, the scope of the truth predicate of the object language £ arches over not only over T+, but also over the languages of the hierarchy. In this strong sense, £ is expressively much richer than T+.

Suppose again that you’re omniscient, and you say “’Snow is white’ is true”. Suppose your task is to lay out the extension of your use of ‘true’. You know that your use of ‘true’ has singularities – it is always possible to produce an anaphoric liar by adding, “but this very sentence isn't". But you know that in other contexts there are true reflections on this liar, and you can throw these true reflections into the extension of your use of 'true'. And since you're familiar with the singularity theory, you know that this anaphoric liar is $\text{true}_{\pounds}$ - that is, true in some context. And this context-independent truth – that the liar sentence is $\text{true}_{\pounds}$ -- can be thrown into the extension too. And so can the truths of the language T+, and the truths of the hierarchy generated from T+. A decontextualized truth predicate doesn't compromise the singularity theory. Rather, it provides more truths to be included in the extension of an ordinary use of 'true'. A given use of 'true' (or 'denotation' or 'extension') is almost global, and applies almost everywhere, except to its singularities. But this expressive incompleteness is minimal, and local. The singularities it fails to evaluate can always be evaluated elsewhere. Our omniscient being can say everything there is to say, but not all at once.

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Endnotes

[1] Kripke, in Martin 1984, p.80.

[2] See Field 2008.

[3] As Field puts it: "If we think of a determinately operator as attaching to a truth predicate to yield a predicate of "strong truth", we can think of the theory as providing an account of hierarchy of "stronger and stronger truth predicates". But unlike most approaches that allow a hierarchy of "truth predicates", no infinite hierarchy of metalanguages is required." (Field 2003, p.140)

[4] To illustrate the point, consider a simple hierarchical view, where truth is stratified into levels. Suppose we adopt such a view. Given a liar sentence

(A) A is not true$_\alpha$,

we suppose that 'true$_\alpha$' is the truth predicate for a language *L* ('true$_\alpha$' applies exactly to the true sentences of *L*), and this predicate is expressible in a metalanguage *M* for *L*, but not in *L* itself. We can say A is not true$_\alpha$ – if it were true$_\alpha$, it would be assessable by the true$_\alpha$-schema, and contradiction would follow. But our assessment of A as not true$_\alpha$ does not lead back to contradiction, because we take it that our repetition of A, along with A, are sentences of *M* that are to be assessed not by the true$_\alpha$–schema, but by the schema appropriate for sentences of *M*. Let this be the true$_{\alpha+1}$ schema, applying to exactly the true sentences of *M*, and expressible not in M but in a further metalanguage. Then both A and our repetition will be true$_{\alpha+1}$, since A is not a sentence of L, and so not a true sentence of L - that is, A is not true$_\alpha$,.

[5] See, for example, Stalnaker 1975, 1978, 1988, 1999.

[6] See Lewis 1979, 1980.

[7] Grosz 1977, and Grosz and Sidner 1986.

[8] Halliday 1967, Chafe 1976, Allerton 1978.

[9] A context can also be *non-explicitly* reflective with respect to an expression. I'll turn to this a little later.

[10] For more on this alternative, see Simmons 1993 (on truth), Simmons 1994 (on denotation), Simmons 2000 (on extensions); and for a recently completed draft of a monograph on a unified account of truth, denotation and extensions, see Simmons (2013 ms.).

[11] Gödel 1944, in Schilpp 1944, p.150.

[12] There isn't space to present the formal theory in detail here. But we can see in in outline how things go. The primary representation of (R) indicates that (R) is to be assessed by the reflective truth$_{rL}$-schema. Call the resulting truth value the *primary value* of (R). The primary value of (R) will depend on the only member of (R)'s determination set, namely (L). Since the primary representation of (L) repeats on the infinite branch of (R)'s primary tree, (L) is identified as a singularity of 'true$_L$'. And so (L) is not true$_L$, just as (R) says. So we can reflectively establish the value *true* for (R). (R)'s primary value is *true* (that is, true$_{rL}$).

[13] It is perhaps worth noting that the primary representation of (R) is a secondary representation of (L).

[14]. Compare Kripke's claim that the 'level' of an ordinary statement involving truth depends on the empirical facts about the statement, and should not be assigned in advance by the speaker: "in some sense a statement should be allowed to seek its own level" (Kripke 1975, in Martin 1984, p.60. See pp.60-1 and pp.71-2.). Setting aside the notion of level (since the singularity approach does not stratify the truth predicate), the case of U is broadly in line with Kripke's claim. The extension of 'true' in U depends on features of the context (the empirical facts about U spelled out by the semantic network U generates in its context of use) that need not be known in advance by the speaker.

[15] It is sometimes objected that hierarchical versions of contextualism about truth have no systematic way to attribute levels to a given utterance. This isn't a complaint that can be leveled against the singularity theory. According to the singularity theory, truth isn't stratified – rather, it's a matter of identifying the singularities of a given occurrence of 'true'. And for that, we need only the notion of the reflective status of a context – either given explicitly as a feature of the context, or given via the semantic connections displayed in primary trees. As we've seen, even if Max produces the Liar sentence (L), this too is assessed by the c$_U$-schema. There is no need to exclude (L), or any of Max's utterances, from the scope of the true$_U$-schema.

Now, since none of Max's utterances are excluded from the scope of the true$_U$-schema, your utterance U is equivalent to the infinite conjunction:

if Max says "S_1" then S_1, and if Max says "S_2" then S_2, and … .

Your use of 'true' in U allows you to express a generalization that it would otherwise take an infinite conjunction to express. Disquotationalists about truth have emphasized the intersubstitutability of "S" and "S is true", and the *expressive* role of truth – its role in expressing generalizations that we couldn't otherwise express. The singularity theory accommodates the expressive role of truth.

But there are limits to the intersubstitutability principle. Consider again the Liar sentence (L) again, represented as

(L) (L) is not true$_L$.

Consider (L) and '(L) is true$_L$'. Consider a reflective evaluation of these. For example, consider the evaluation of these by the true$_{rL}$-schema. (L) is true$_{rL}$. But '(L) is true$_L$' is false$_{rL}$ – it's not the case that (L) is true$_\alpha$ (it fails to have true$_\alpha$ conditions). So here the intersubstitutability principle fails. This is related to a point made in Dummett 1959, p.145.

So we need to restrict the truth-schema. Once we have the notion of a singularity on board, we can provide a minimally restricted truth-schema for an occurrence of ‘true’ tied to context α:

If ‘S’ is not a singularity of ‘true_α’, then ‘S’ is true_α iff S.

Singularities aside, intersubstitutability holds. Similarly for denotation and extension.

[16] See Priest (2006), especially Chapter 20, and footnote 12.

[17] There is more to say about the singularity theory and revenge that I can say here – see Chapter 12 of Simmons 2013 for an extended discussion.

[18] This is a convenient way to talk -- but it should be understood that to say, for example, that (L) is true in (R)’s context of use is a shorthand way of saying that it’s true when assessed by the schema associated with (R)’s context of use.

[19] Recanati (2002), p.312.

[20] Jason Stanley takes certain contextualist accounts of truth, such as that in Burge 1979, to hold that ‘true’ is a narrow indexical, and he argues that this claim is put into doubt by the observation that the vast number of cases of unobvious context dependence do not involve narrow indexicality – see Stanley 2000. But the singularity account does not take ‘true’, denotes’ or ‘extension’ to be narrow indexicals.