Chapter 9

Revenge, II

9.1 Contextual theories and direct revenge

In the last chapter, I distinguished two kinds of revenge: direct and indirect. In the case of direct revenge, the semantically defective sentence or expression is reinstated by the very attempt to declare it defective. As we've seen, direct revenge is a problem for a number of theories of truth. But the singularity account provides a natural treatment of direct revenge.

In one manifestation, direct revenge is just the phenomenon of *repetition*: for example, in the case of C ('the sum of the numbers denoted by expressions on the board'), we repeat the very words of C to produce C*, but in a context reflective with respect to C – and in this context, C* is evaluated by a reflective r_C-schema, and we find that when C* is reflectively evaluated, it denotes π+6. In another manifestation, direct revenge is just the phenomenon of *rehabilitation*: we revisit the expression C itself on the board, and assess it reflectively by an r_C-schema, which yields the denotation π+6. Far from being a serious stumbling block, the phenomenon of direct revenge is, for the singularity theory, a major motivation.

Iteration is another manifestation of direct revenge, and this is also readily handled by the singularity theory. Take the case of C. C and C* are both represented by 'the sum of the numbers denoted$_{cC}$ by expressions on the board'. As we saw in Chapter 2, our reflective re-evaluation R of C may be represented as follows:

(R) The phrase C denotes$_{cR}$ π+6, where c_R is R's context of use, a context reflective with respect to C, and 'denotes$_{cR}$' represents any occurrence of 'denotes' coextensive with the occurrence in R. We can now reason on from (R):

(#) The sum of the numbers denoted$_{cR}$ by expressions on the board in room 213 is п+6+(п+6).

As we observed in Chapter 2, (#) contains a token C** of the same type as C, and C^{**} does denotes$_{cR}$ π+6+(π+6). But C** is a token distinct from C. C does denotes$_{cR}$ π+6, but it doesn't also denote$_{cR}$ π+6+(π+6). And C^{**} is not an expression on the board, so the iteration is halted. The apparent iteration only gets off the ground by confusing C with another token of the same type. In Chapter 2, we also saw how the singularity theory handled iteration in the cases of the extension and truth.

The phenomenon of direct revenge motivates not only the singularity theory but other contextual theories, in particular those of Burge, Parsons, Gaifman, Glanzberg, and Barwise and Etchmendy.[1] These contextual theories are focused on truth, and for all of them, some version of the reasoning that generates *repetition* and *rehabilitation* is crucial. Here is one version. We start with a Liar sentence:

(L) L is not true.

We reason in the usual way to the conclusion that (L) is defective or pathological, and so infer:

(M) L is not true.

Since L and M say the same thing, and M is true, we conclude:

(N) L is true.

The singularity theory identifies L as a singularity of the occurrence of 'true' in L (and in M), but not of the occurrence of 'true' in N. But the occurrence of 'true' in N has its own singularities, and the extension of 'true' in N is neither more nor less comprehensive than the extension of 'true' in L. There is no stratification of the truth predicate.

In contrast, other contextual theories discern a hierarchical shift in levels between L and N. According to Burge, the occurrences of 'true' in L and 'true' in N correspond to distinct levels in a Tarskian hierarchy.[2] We can think of 'true' as it occurs initially in L as indexed to a certain level represented by the number i.[3] So we can represent the occurrence of 'true' in L as 'true_i', where the numerical subscript indicates the level fixed by the context. And when we 'rehabilitate' L, and declare it true, we do so at a higher level than i, say k: (L) is not true_i, but it is true_k, where $k>i$.[4]

Both Burge's Tarskian theory and the singularity theory locate the context-sensitivity associated with the liar in the truth predicate – 'true' shifts its extension according to context. Another contextualist approach locates the context-sensitivity elsewhere, in a more general setting not limited to truth. If I say "There's no beer left", I do not mean there is no beer left in the entire world – I mean there's none left in the refrigerator. Context determines the domain of quantification here. The idea that the context-dependence of truth is derived from the context-dependence of quantifier domains was first suggested by Parsons and developed in a fully rigorous way by Glanzberg.

Parsons suggests that we can accommodate the conclusion of our strengthened reasoning, that L is true, if we assume that this evaluation "presupposes a more comprehensive scheme of interpretation than the discourse up to that point".[5] The final evaluation "involves a semantical reflection that could be viewed as involving taking into one's ontology a proposition that had not been admitted before."[6] Glanzberg makes these ideas precise in Glanzberg 2001 and Glanzberg 2004.

Glanzberg takes *propositions* to be the bearers of truth, and so we should consider this version of the Liar:

(L*) L* does not express a true proposition.

This yields a revenge liar in terms of propositions. In the course of the Liar reasoning, we can show that on pain of contradiction that

(a) L* does not express a proposition.

So it follows that

(M*) L* does not express a true proposition.

But this is just L* again. (This may be regarded as a propositional version of *Repetition*.) So we have proved L* – and so L* is true. (This is *Rehabilitation*.) But for a sentence to be true is just for it to express a true proposition. It follows that:

(b) L* does express a proposition.

(a) and (b) are contradictory, and we are landed in paradox.[7]

Glanzberg focuses on (a) and M*. Both are true, because both are established by sound reasoning. But since (a) is true, there is no proposition expressed by L*. But if M* is true, then there is a proposition expressed by L*. We *proved* that L* cannot express a proposition, and then we *proved* that L* can. But how can it be that L* at first fails to express a proposition, and then succeeds? Without admitting the context-dependence of the liar sentence, the question seems unanswerable. According to Glanzberg, we are forced to conclude that L* exhibits some context-dependence. Glanzberg argues that since propositions are the truth bearers, and the truth values of propositions do not vary from context to context, the predicate ‘true’ will not itself be context-dependent.   But things are different with the expression relation: it is perfectly possible for a sentence to express a proposition in one context but not in another. So we have a way of answering the question: in the context of (a), there is no proposition expressed by the liar sentence L*, but in the context of M*, there is. There is a shift in the domain of the propositional quantifier. The domain of propositions associated with the context of (a) does not contain a proposition for L* to express; the expanded domain associated with the context of M* does.[8]

For Glanzberg and Parsons, the key to a solution to the revenge liar is the context dependence of quantifier domains – versions of *Repetition* and *Rehabilitation* are explained by a contextually determined expansion of the background truth conditions. Barwise and Etchemendy also explain revenge in terms of a contextually determined expansion – but of *situations*, not quantifier domains. Barwise and Etchemendy employ two main tools, the notion of a situation taken from situation semantics,[9] and Aczel's set theory,[10] which provides a set-theoretical way of modeling circular propositions. The upshot of their theory is that we can think of the liar sentence as providing a propositional function that 'diagonalizes out' of any set of propositions. In this way, a version of *Repetition* is explained. Consider a Liar sentence, say,

(λ) The proposition expressed by λ is false,

According to Barwise and Etchemendy's analysis, the proposition expressed by the sentence λ is false. So we can step back and recognize the falsity of (λ), consider that new fact, and say:

(μ) The proposition expressed by λ is false.

If we suppose that the proposition expressed by λ is about the situation s_1, then the fact that the proposition expressed by λ is false cannot be in s_1. But μ expresses a different proposition, since this proposition is about a different *extended* situation s_2, where s_2 is the result of adding to s_1 the fact that the proposition expressed by λ is false. And so the proposition expressed by μ is true. Both λ and μ are of the same type, but they are about different situations. This version of the revenge liar is resolved by a contextually determined shift in what the two sentences are about –

the domain of facts expands. Because of this expansion, μ can truly say something that λ cannot.[11] There is an analogous Liar proposition about the situation s_2, which leads to a more comprehensive situation s_3, and so on, through a hierarchy of expanding situations.

9.2 Contextual theories and second-order revenge

It is an advantage of contextual theories that they handle direct revenge so naturally. Any theory of truth must say that the liar is defective in some way – but then a revenge liar that says of itself that it's defective in this way will be true. Contextual theories meet this challenge because they recognize that we can reason through pathology: assessment of the liar sentence breaks down in the initial context, but goes forward in the reflective context, just because it broke down initially.

While direct revenge provides motivation for contextual theories, second-order revenge presents challenges. Consider Burge's Tarskian account, according to which any occurrence of 'true' is tied by context to a particular level of language. Then it may seem that a paradox is generated by the sentence 'This sentence is not true at any level'.[12] Burge responds that the attempt here to produce paradox is misguided – it tries, and inevitably fails, to 'de-indexicalize' 'true'. Even in the phrase 'true at some level', there is an implicit index on 'true', so the attempt to quantify out the indexical character of 'true' leads to incongruity. Compare this with my saying on an occasion that 'I'm hungry now at some time' or "I'm here at some place".[13] According to Burge, 'true', like the indexicals 'now' and 'here', is not bindable by operators.[14]

A related challenge is posed by statements such as "All sentences are true or not" – how can such a global statement be accommodated if a use of 'true' is always tied to a definite level? If we take such a statement to be asserted in a particular context, with a particular index on 'true', then the broader import of the statement is compromised. In response, Burge distinguishes between *indexical* and *schematic* uses of 'true'. A predicate on an occasion of use is *indexical* if its extension depends on the context of use; it is *schematic* if it doesn't have a definite extension on that occasion, but through its use on that occasion provides general systematic constraints on the extension of the predicate on other occasions of use. Burge takes the formal principles of his theory to be stated schematically – and they are to be evaluated as true, where 'true' is being used schematically.[15] Likewise, the global statement above is a schematic generalization. Its formalization is: $(s)(Tr_i(s) \vee \sim Tr_i(s))$, where the subscripts stand open, ready to be filled in as the occasions arise. And when we evaluate this schematic statement as true, we are using 'true' schematically. As Burge points out, schematic uses are not mysterious – they're mathematically well-entrenched and useful.[16] Genuine revenge cannot set aside the distinction between indexical and schematic uses.

Since Glanzberg's theory resolves the liar via an expansion of the quantifier domain, a *universal* domain of quantification cannot be admitted. Glanzberg argues for the general claim that there are no absolutely unrestricted quantifiers. It is an advantage of Glanzberg's approach that this response is independent of special considerations about the liar, and so escapes any charge of adhocness. Glanzberg takes it as truistic that meaning is a matter of interpretation, and that interpretation must provide a domain of quantification. The key question is whether it is possible for a speaker to specify a domain of 'absolutely everything'. Usually domains are specified by using predicates, but what predicate could specify a universal domain? The predicate 'object' might be suggested, but this predicate seems too vague to yield a determinate domain, and does not provide, by itself, a preferred sharpening (even a nominalist will not claim that it's part of the *meaning* of 'object' that objects are concrete). The best hope for a maximally broad conception of object, Glanzberg suggests, is found in logic, via the logical notion of a singular term: an object is whatever a singular term refers to. Glanzberg gives his opponent this logical notion of object, but argues that it is still impossible to specify a domain of absolutely all objects – because of Russell's paradox. Given a specification of a domain, we can quantify over it and form the class term '{x:x=x}'. The class {x:x=x} cannot be in the domain over which x ranges, since if it were, the Russell set {x:x∉x} would be in the domain, by (restricted) comprehension. So we can never specify a domain of absolutely everything – something is always left out. And the argument need not be in terms of classes or sets: a general version of Russell's paradox can be formulated in terms of the notion of *interpretation*.[17] The process of interpretation itself can take us beyond the domain of any interpretation we produce. The logical notion of object is indefinitely extensible.

The claim that that there are no absolutely unrestricted quantifiers is in line with most quantification in natural language. Quantifiers are usually restricted by predicates ("There's no beer in the fridge') or by the context ("Everything was destroyed in the fire"). Moreover, the extensibility of the logical notion of object is a special sort of expansion – the additional objects are artifacts of the process of interpretation, so that this expansion of the background domain has little practical effect on what we say. There are, however, cases where it might seem counterintuitive to give up on absolutely unrestricted quantification. Consider, for example, a logical truth such as "All objects are self-identical". Glanzberg argues that this logical truth seems to be about absolutely everything because its truth does not depend on what the domain is. Though its quantifier must be interpreted as ranging over some domain, it tells us something beyond this: it tells us, in an ambiguous way, something about any domain it might be interpreted as ranging over. Such statements exhibit *typical ambiguity*: though its meaning is still fixed by its interpretation, with a specified domain, we can recognize that it would hold whatever domain was specified.[18] Logical validities provide one case of typical ambiguity. Global semantic statements provide another: any utterance of 'Every proposition is true or not' comes with a contextually determined domain of truth conditions, but we can see that the statement would hold whatever the background domain. These special cases carry additional force – but that's because they're typically ambiguous, and not because they are genuine cases of unrestricted quantification.[19]

Just as Glanzberg argues that we cannot specify a domain of all objects, or all propositions, so Barwise and Etchemendy claim that no proposition can be about the world as a whole. Though (Austinian) propositions can be about extremely comprehensive situations, the falsity of a liar proposition, though a feature of the world, cannot be a feature of the situation the proposition is about. There is a hidden parameter that the Austinian account of the liar makes explicit: the part of the world that the proposition is about. Now in general the boundaries of the situation a person is referring to may well be unclear. Barwise and Etchemendy argue that this vagueness injects ambiguity into the everyday use of language – it is easy to think that the falsity of the liar proposition is part of the situation that the liar is about (contrast λ and μ above). This can make the liar seem intractable. But once we take proper account of these boundaries, we can draw the lesson of the liar: we cannot make statements about the whole world, about the universe of all facts.[20]

There are some important lessons to be drawn from these responses to revenge. In particular, there is the crucial point that generalities can be expressed by schematic -- or typically ambiguous – statements. But unlike the contextual theories just surveyed, the singularity theory does not appeal to a hierarchy – of truths, or domains of propositions, or of situations. Instead it places minimal restrictions on occurrences of ‘denotes’, ‘extension’ and ‘true’, minimal restrictions that shift with the context, and allow the application of the truth predicate to be as close to global as possible. Contextual-hierarchical theories will face challenges in part because they are hierarchical solutions. Why think that a natural language such as English is stratified in this way? Why can’t we quantify over all levels of Burge’s hierarchy, for example, and regenerate paradox? (Think of the parallel pressure to quantify over all levels of ZF set theory.) Why should we accept Glanzberg’s restrictions on quantification, or Barwise and Etchemendy’s claim that we cannot talk about the whole world? These are hard questions for contextual-hierarchical theories, and the threat of revenge persists. But the singularity theory doesn’t stratify ‘true’ or ‘denotes’ or ‘extension’, and so we might reasonably expect that it’s immune to the kind of revenge aimed at hierarchical accounts. Perhaps, then, the singularity account offers something new when it comes to second-order revenge – and it’s to this possibility that I now turn.

9.3 The singularity theory and revenge

To fix ideas, recall from Chapter 6 that *L* is a fragment of English that contains no semantic terms (and, further, for the sake of simplicity, no context-sensitive terms or vague terms). We obtain the language ₤ by adding to *L* the term *t*, where *t* is one of the terms 'denotes' or 'extension' or 'true' or 'true of'. (For convenience, I shall use ‘₤’ to pick out each of the languages *L*+‘denotes’, *L*+‘extension’, or *L*+‘true’, or *L*+‘true of’ – it should be clear at any given point which of these extended languages is under discussion.) ₤ is the language that the singularity theory is a theory of. In what follows, a key claim that will emerge is this: the language in which the singularity theory is couched – call it $\mathfrak{I}$ – is *not* a Tarskian metalanguage for £.

The conceptual resources to which the singularity theory appeals are remarkably meager, as I noted in 6.7. Let's review them now.

(1) The theory must be able to provide the *primary representations* of every expression of £. Suppose σ is an expression of £. If σ is not a repetition or a variant repetition, then the primary representation of σ is $\langle \mathrm{type}(\sigma),c_{\sigma},c_{\sigma}\rangle$. For this, then, the theory must be able to make reference to the *type* and the *context* of any given expression of £. If σ is a repetition or a variant repetition, we will need to import the explicitly reflective character of a context into the formal theory – for example, if σ is a repetition of ρ, then its primary representation is $\langle \mathrm{type}(\rho),c_{\rho},r_{\rho}\rangle$. So here we need the additional notion of a context that is *explicitly reflective with respect to a given expression*.

(2) The theory must be able to provide *secondary representations* of every expression of £. If σ is not a repetition, then a secondary representation of σ has the form $\langle \mathrm{type}(\sigma),c_{\sigma},\alpha\rangle$, where α is a context other than c_{σ}. If σ is a repetition, say of ρ, then a secondary representation of σ has the form $\langle \mathrm{type}(\rho),c_{\rho},\alpha\rangle$, where α is a context other than r_{ρ}. And if σ is a variant repetition of ρ, then a secondary repetition of σ has the form $\langle \mathrm{type}(\rho),c_{\rho},\alpha\rangle$, where again α is a context other than r_{ρ}. In all these cases, the notions required are just the ones already covered: the context of a given expression of £, and the notion of a context reflective with respect to a given expression.

(3) The theory must be able to accommodate *trees*, both primary and secondary. For these, we need the notion of a *determination set*, the set of expressions to which a given expression makes reference. The theory then provides a systematic way of moving down the branches of a tree. Suppose a node of a tree represents the expression τ, and suppose υ is in τ's determination set.

If c_τ is not explicitly reflective with respect to υ, then the node for υ immediately below the node for τ will be $<\text{type}(\upsilon),c_\upsilon,c_\tau>$. If c_τ is explicitly reflective with respect to υ, then the node for υ will be $\text{type}(\upsilon),c_\upsilon,r_\upsilon>$. Trees utilize the same resources as before, together with the notion of a determination set.

(4) These resources are all we need to define the notions of a *0-expression*, a *pruned*$_0$ *tree*, a *loop*, a *chain*, a *reflection-free* expression, a *1-expression*, a *pathological* reflection-free expression, a *key singularity*, and a *determination tree* for a pathological reflection-free expression. And the same is true for the higher levels of the reflective hierarchy, through the notions of a *pruned*$_\beta$ *tree for an expression* σ, a β*-reflective expression*, a *pruned*$_{<\alpha}$ *tree***,** a $<\alpha$*-reflective expression***,** an α*-expression*, a *pathological* α*-expression*, to the notion of a *key singularity* and a *determination tree* for an α-expression.

To sum up, we can identify the key singularities and the determination trees for expressions of £ if we are given just the following: (1) the notion of the type of an expression of £, (2) the notion of an expression's context of utterance, (3) the information that a context is explicitly reflective with respect to a given expression, and (4) the determination set of a given expression.

The relatively meager resources of the theory bear on the issue of second-order revenge. The formal language $\mathcal{T}$ of the singularity theory, in which we define the notions of *primary tree*, *key singularity*, *determination tree*, and the rest, is not a metalanguage for £. Nowhere does $\mathcal{T}$ utilize a predicate coextensive with any occurrence of *t* in £, or anything that could count as the denotation predicate (or extension predicate, or truth predicate) for £. For example, consider the primary representation of C, $<\text{type}(C),c_C,c_C>$. The second element indicates that the occurrence of 'denotes' in C is to be represented by 'denotes$_{cC}$', a representation that indicates merely an occurrence of ‘denotes’ that is coextensive with the occurrence of ‘denotes’ in C. As we stressed in Chapter 2, the subscript indicates nothing specific about the extension of ‘denotes’ in C (in contrast with, say, a subscript that indicates a level in a hierarchy of languages). The third element indicates that C is to be evaluated by the c_C-schema – the schema where the occurrence of ‘denotes’ on the left-hand-side is coextensive with the occurrence of ‘denotes’ in C. So we all we need to specify the primary representation of C is the type of C and the context of C. We don’t need a denotation predicate in the theory ℑ coextensive with ‘denotes’ in C. In general, the theory identifies singularities without in any way ‘modeling’ ₤. A ‘Tarskian’ account of *t* stratifies *t*, providing extensions of *t* at distinct levels; in Kripke’s theory, the minimal fixed point is a model of the object language, providing the extension and anti-extension of ‘true’. There is nothing analogous in the language ℑ of the singularity theory.

In treating the semantic paradoxes along singularity lines, we do not aim to absorb the expressive scope of the object language. Rather, the aim is to formally describe the effect of context on uses of our semantic expressions – more specifically, the aim is to identify pathology and singularities in a rigorous and systematic way. This approach to the paradoxes should be sharply distinguished from another kind of approach, where the strategy is to present a contradiction-free formal language that is supposed to model a natural language like English. For example, Kripke’s theory of truth presents the language of the minimal fixed point as a model of a substantial portion of English. For an account such as Kripke’s, the aim of the theory is to treat semantic paradox (in this case, the Liar) by constructing a consistent language that formalizes a significant portion or stage of English that contains the predicate ‘true’.[21] In contrast, the singularity theory does not provide a model of English, but instead a formal, abstract description of the way in which the extension of *t* depends on context.

So we can identify singularities of a given occurrence of *t* without the ascent to a metalanguage. This does not yet provide a positive account of denotation (or truth, or extension, or truth-of) – identifying singularities tells us what to leave out of the extension, not what to put in. But once we've identified the singularities of an occurrence of 'denotes', 'extension' or 'true' or 'true of', we can take the positive step we need: we can provide schemas for denotation, extension and truth, as we saw in 4.1. Given a context α, the *denotation schema* for expressions of ₤ is this:

(i) If e is not a singularity of 'denotes_α', then e denotes_α k iff d=k, and

(ii) if e is a singularity of 'denotes_α', then e does not denote_α,

where d is replaced by a denoting expression, and e is replaced by a name of that expression.

The extension schema:

(i) If φ is not a singularity of 'extension_α', then for all x, x is in the extension_α of φ iff x is Φ, and

(ii) if φ is a singularity of 'extension_α', then φ has no extension_α,

where x arranges over objects, Φ is replaced by a predicate, and φ is replaced by a name of that predicate. And the truth schema:

(i) if s is not a singularity of 'true_α', then s is true_α iff S, and

(ii) if s is a singularity of 'true_α', then s is not true_α (or false_α),

where S is replaced by a sentence, and s is replaced by a name of that sentence.

We are now moving beyond the confines of the singularity theory. The singularity theory provides the following diagnosis of the semantic paradoxes: our semantic expressions are context-sensitive, and their application breaks down at certain points, the singularities. The theory also provides a rigorous way to identify the singularities of a certain occurrence of 'denotes', 'extension', 'true' or 'true of' in the language ₤, and a recipe for determining the semantic value of expressions of ₤.

However, it is natural to go further and provide glosses on the theory that make the context-sensitivity explicit. We can say things such as 'C does not denote when evaluated by the unreflective c_C-schema' and 'C does denote when evaluated by a reflective r_C-schema'. In Chapter 2, we drew parallels between the context-sensitivity of 'denotes' and the expressions 'hexagonal' and 'flat'. If α is a context in which it's true to say that Italy is boot-shaped, then in that context my utterance of 'France is hexagonal' will be true. Here 'hexagonal' is a context-sensitive term. But we can make the context-sensitivity explicit, as in the 'decontextualized' sentence "France is hexagonal by the standards of precision operative in context α". Similarly, when I write the token C on the board, my use of 'denotes' is context-sensitive. It is sensitive to the standards of assessment that are operative in its context of utterance, namely a schema, the c_C-schema, that is unreflective with respect to C. But we can make the context-sensitivity explicit, as in 'the sum of the numbers denoted by expressions on the board when they're evaluated by the unreflective c_C-schema'. The fact that we can decontextualize in this way does not compromise the context-sensitivity of 'hexagonal', 'flat', or 'denotes'. Compare again Recanati on expressions like 'small' which trigger a comparison class:

> "When the comparison class is made explicit, as in 'Sherman is small for a basketball player', the denotation of the complex predicate ('small for a basketball player') is independent from extralinguistic context, and the expression 'small' no longer functions as an indexical. It is only when the comparison class is left implicit, as in 'Sherman is small', that the property denoted by the predicate is denoted in a context-dependent manner: the predicate may then still denote the property of being small for a basketball player, but it denotes the property only with respect to a context in which the class of basketball players is made salient as the relevant comparison class".[22]

So the terms ‘denotes’, ‘extension’, and ‘true’ are like ‘hexagonal’, ‘flat’ and ‘small’ in that they can be used as context-sensitive expressions, or as part of decontextualized complex terms.

If we accept that the ‘narrow’ indexicals – ‘here’, ‘now’, ‘I’ – are nonbindable, then ‘denotes’, ‘extension’, ‘true’ are not *narrow* indexicals. Given the singularity theory, it is natural to formulate claims that introduce binding operators, as in ‘C denotes when evaluated by some schemas but not by others’. It seems to me that it’s implausible for a contextualist to disallow such locutions – it seems an *ad hoc* maneuver to avoid the perceived threat of revenge. It is a disadvantage of Burge’s account that ‘true’ is treated as a narrow indexical, so that decontextualized phrases such as ‘true in some context’ are regarded as incongruous.[23]

We are now in a position to formulate decontextualized predicates associated with revenge paradoxes. We can, for example, form the following predicate applied to expressions of £**:** ‘expression of £ that, for some context α, denotes$_{\alpha}$’. This 1-place predicate, ‘denotes$_{£}$’ for short, may be regarded as a denotation predicate for £, since it applies to all the expressions of £ that do denote when evaluated by an α-schema for some context α -- it applies not only to expressions like ‘London’ and ‘the square of 3’, but to expressions, like C, which denote a particular number when evaluated by some suitably reflective schema. We can also form the 1-place predicate ‘denoted$_{\alpha}$, for some context α, by an expression of £’, or ‘denoted$_{£}$’ for short. With ‘denoted$_{£}$’ on board, we can form expressions associated with revenge paradoxes such as ‘the least ordinal not denoted$_{£}$’. If we switch £ so that it’s the object language containing ‘extension’ rather than ‘denotes’, we can introduce the predicate ‘extension$_{\alpha}$ of a predicate of £ for some context α’, associated with revenge expressions such as ‘non-self-membered extension$_{£}$’. And similarly we can introduce the predicate ‘true$_{£}$’ -- ‘sentence of £ that is true$_{\alpha}$ for some context α' -- where an associated revenge sentence is ‘This sentence is not true$_{\text{£}}$’, or, spelled out, ‘This sentence is not a true$_{\alpha}$ sentence of £ for any α’.

Let’s start with the case of denotation, and consider the denotation predicates for £, ‘denotes$_{\text{£}}$’ and ‘denoted$_{\text{£}}$’. The cases for extension and truth will run parallel. As we saw in Chapter 4, every occurrence of the context-sensitive predicate 'denotes' of £ has singularities (created, for example, by perverse continuations such as “plus the sum of the numbers denoted by phrases in this sentence”) – and these singularities do denote$_{\text{£}}$, since they are expressions of £ that denote when evaluated from appropriately reflective contexts. So in this way the predicate 'denotes$_{\text{£}}$' will have within its scope expressions beyond the scope of any given occurrence of the predicate ‘denotes’ of £ (more precisely, given any occurrence of ‘denotes', there will be expressions to which ‘denotes$_{\text{£}}$’ applies which are not the first member of any ordered pair in the extension of that occurrence). And similarly for 'denoted$_{\text{£}}$', since the phrase ‘the least ordinal not denoted$_{\text{£}}$’ denotes an ordinal beyond the scope of all phrases of £.

This may tempt us to suppose that ‘denotes$_{\text{£}}$’ and ‘denoted$_{\text{£}}$’ are more comprehensive than any occurrence of 'denotes' in the object language £. To put the temptation another way, suppose we add the predicates ‘denotes$_{\text{£}}$’ and ‘denoted$_{\text{£}}$’ to the language $\mathfrak{T}$ of the singularity theory. We may be tempted to suppose that the resulting language – call it $\mathfrak{T}$* -- is a Tarskian metalanguage for £. But this temptation should be resisted.

The scope of ‘denotes$_{\text{£}}$' is restricted to the expressions of £. In contrast, a given occurrence of 'denotes’ applies to all denoting expressions except its singularities. It applies to any denoting expression of any language, as long as the expression is not identified as a singularity. In the language $\mathfrak{T}$*, we can formulate denoting expressions that cannot be formulated in £ -- any denoting expression that uses ‘denotes$_{\text{£}}$’ or ‘denoted$_{\text{£}}$’ is like this. An uninteresting example is 'the least positive integer denoted$_{₤}$', which, like many phrases of ₤, denotes 1. An interesting example, related to revenge, is the König-like phrase K:

(K) the least ordinal number not denoted$_{₤}$.

If this phrase does denote an ordinal, it denotes an ordinal different from any ordinal denoted by a phrase of ₤.[24] But, by Minimality, these denoting phrases, interesting and uninteresting, will *not* be excluded from the extension of any occurrence of 'denotes' in ₤. Only the singularities of the given occurrence are excluded -- and unproblematic denoting expressions expressed in $\mathfrak{I}^*$ will not be identified as singularities.

Imagine an omniscient being in room 213 observing as I write on the board the expressions A ('pi') , B ('six') and C ('the sum of the numbers denoted by expressions on the board in room 213'). The being knows all about the context-sensitivity of 'denotes', the language $\mathfrak{I}^*$, the phrase K, and so on. Our being can survey all the successful denoting phrases of ₤ and of $\mathfrak{I}^*$. The being knows that the pathological token C can be repeated and rehabilitated in suitably reflective contexts. And it recognizes that the decontextualized expression K denotes an ordinal – and so it can place K in the extension of the token of 'denotes' that I write on the board. (Just as the being knows French, and can place a French denoting phrase in the extension of 'denotes', even though the phrase is not expressible in ₤, so the being can place K in the extension of 'denotes', even though K is not expressible in ₤.) No contradiction arises from this instance of the c_C-schema:

$$\text{K denotes}_{cC}\ \delta \text{ iff } \delta = \text{the least ordinal number not denoted}_{₤},$$

and our omniscient being can identify the ordinal δ. But not even an omniscient being can place the singularity C in the extension of the occurrence of 'denotes' on the board.

So ℑ* is not a Tarskian metalanguage for £, since ordinary context-sensitive uses of 'denotes' apply to denoting expressions that can be formulated in ℑ* but not in £, including those that contain the predicates 'denotes$_{£}$' or 'denoted$_{£}$'. According to the singularity proposal, paradox is avoided not by a Tarskian ascent, but by the identification and exclusion of singularities.[25]

Further, ℑ* is in certain respects expressively far weaker than £. ℑ* is a 'scientific' language in which we describe the semantics of a context-sensitive term. Using the language ℑ*, we take context-sensitive language to be the object of our study, and stand back from the contexts and the context-sensitive term that we are describing. We formally define notions like *primary tree* and *singularity*. In describing the behavior of the context-sensitive term 'denotes' (or 'extension' or 'true'), we do not use context-sensitive terms. There are no context-sensitive terms in ℑ*. When we present the singularity theory, we take up an abstract, theoretical point of view. ℑ* is not a language that contains an expression tied to context - it is *about* a language that contains an expression tied to context.

But since ℑ* is a classical scientific language free of context-sensitive terms, vagueness, ambiguity, and so on, it is provably subject to expressive limitations. In particular, ℑ* cannot contain *its* own denotation predicate (or extension predicate, or truth predicate). Let 'denotes$_{\mathfrak{I}*}$' be the denotation predicate for ℑ*, given by:

'e' denotes$_{\mathfrak{I}*}$ x iff e is an expression of ℑ* and e=x.

We can then form the 1-place predicate 'denoted$_{\mathfrak{I}*}$' which applies to exactly those objects which are the second members of the ordered pairs in the extension of 'denotes$_{\mathfrak{I}*}$'. If we suppose that 'denotes$_{\mathfrak{I}*}$' is a predicate of ℑ*, then a version of König's paradox is generated by the phrase ''the least ordinal not denoted$_{\mathfrak{I}*}$'. On pain of contradiction, the predicate 'denoted$_{\mathfrak{I}*}$' is contained in a metalanguage for $\mathfrak{I}$*. And the metalanguage for $\mathfrak{I}$* is subject to the same kind of expressive limitation, and we are led to a hierarchy of languages with $\mathfrak{I}$* at its base, each containing a denotation predicate for the preceding language. But none of the denoting expressions expressible in these languages are identified as singularities of an occurrence of 'denotes' in ₤, even if they contain 'denotes$_{₤}$' or 'denoted$_{₤}$' or 'denotes$_{\mathfrak{I}*}$' or 'denoted$_{\mathfrak{I}*}$', and so on, through the denotation predicates of the hierarchy. And so none of these expressions are excluded from the extensions of our ordinary context-sensitive uses of 'denotes'. To speak metaphorically, our uses of 'denotes' (and 'denoted') in ₤ arch over not only the denoting expressions of $\mathfrak{I}$*, but also over the denoting expressions expressed by the languages of this hierarchy. Our omniscient being, in front of the board in room 213, can survey all the successful denoting expressions of this hierarchy, and place them in the extension of the occurrence of 'denotes' on the board.

We can introduce a broader denotation predicate applying not just to the denoting phrases of ₤ (a fragment of English), but to denoting phrases in other natural languages. We can consider *analogues of ₤*: fragments of other natural languages, composed of the non-semantic part of each language (the analogues of *L*) plus the translations of 'denotes' (or 'extension' or 'true'). We can then form a denotation predicate applying to all denoting expressions of *₤* and its analogues: 'is an expression of *₤* or an analogue of *₤* that denotes$_\alpha$ for some context α'. And we can form a predicate applying to the objects that are denoted by these expressions: 'denoted$_\alpha$, for some context α, by an expression of ₤ or an analogue of *₤*'. Now we can form the Konig-like phrase K^+:

(K^+) the least ordinal not denoted$_\alpha$, for any context α, by any expression of ₤ or analogue of ₤.

Assuming that this phrase does denote an ordinal, it denotes an ordinal beyond the scope of any expression of ₤ or any analogue of ₤. But still, according to the singularity theory, the phrase is within the scope of any occurrence of ‘denotes’ or its translation into some other natural language. Our omniscient being in room 213 knows all about K^+ too.

The crucial distinction here is between languages like £ and its analogues that contain a context-sensitive term, and ‘decontextualized’, theoretical languages like 𝔗 and 𝔗*. Context-sensitive uses of ‘denotes’ in £ fall short, minimally, of universality – each occurrence has singularities, as we’ve seen. But any decontextualized language, such as 𝔗 or 𝔗*, must also fail to contain a global denotation predicate. For example, as we’ve noted, the denotation predicate for 𝔗*, ‘denotes_{T*}’, is expressible only in a metalanguage for 𝔗*. This metalanguage contains a denotation predicate for 𝔗*, but not for itself – that requires a further metalanguage. No decontextualized language can contain a global denotation predicate, ‘denotes_{G}’, on pain of the contradiction generated by the Konig-like phrase ‘the least ordinal not denoted_{G}’.

Now there are theoretical, decontextualized languages that contain a predicate that applies to all the expressions of £ that denote in some context, or, further, to all the expressions of £ and its analogues that denote in some context. In this respect, the decontextualized languages are expressively more powerful than £ and its analogues: they contain phrases that denote ordinals not denoted in any context by any expression of £ or an analogue. But each of these decontextualized languages cannot contain a denotation predicate with scope over its own denoting expressions or those further up the hierarchy. However, *these* expressions *are* within the scope of any use of ‘denotes’ in £. An occurrence of ‘denotes’ fails to apply only to its singularities, and so it does apply to the denoting expressions of any decontextualized language. In this respect, natural language fragments like £ and its analogues are expressively far more powerful than any decontextualized, theoretical language.

So no classical decontextualized language can contain a predicate that matches the scope 'denotes' in ₤, since the denotation predicate in the language cannot have scope even over its own denoting expressions (or those further up the hierarchy). Consider a theoretical, decontextualized language $\mathfrak{I}_\delta$, where δ indicates its level in the hierarchy generated from $\mathfrak{I}^*$. Let 'denotes$_\delta$' represent the denotation predicate whose extension comprises the denoting expressions of $\mathfrak{I}_\delta$. This denotation predicate will be expressible in a metalanguage $\mathfrak{I}_{\delta+1}$ for $\mathfrak{I}_\delta$. Any occurrence of this predicate in $\mathfrak{I}_{\delta+1}$ will be tied to its level, and apply only to the denoting expressions of the languages at lower levels. The formation of a predicate that tries to escape this restriction cannot work. For example, consider the predicate 'denoted$_\delta$ by any expression of the hierarchy'. The last six words are idle; they cannot override the level at which the denotation predicate is fixed. Similarly with the predicate 'denoted$_\delta$ by some expression in some language in some context'. The attempt at a global reach fails. And so the corresponding attempt at another version of revenge fails. The expression 'the least ordinal not denoted$_\delta$ by any expression in any language in any context' does not generate a paradox. No paradox arises from the supposition that there is a least ordinal not denoted$_\delta$. And the ordered pair of the expression 'the least ordinal not denoted$_\delta$' and the ordinal it picks out (if it does) will be in the extension of any use of 'denotes' in ₤. No decontextualized denotation predicate can be global, or can match the scope of an ordinary use of 'denotes' in ₤.

Indeed, natural language fragments like ₤ are so expressively powerful that no denoting phrase is beyond their reach. Any denoting phrase expressible in the language ₤ is in the scope of some occurrence of 'denotes' – in particular, as we've seen, a singularity of a given occurrence of 'denotes' is in the extension of other occurrences of 'denotes' in suitable (explicitly or non-explicitly) reflective contexts. And as we've just seen, any denoting phrase in a 'decontextualized', theoretical language is in the extension of any occurrence of 'denotes' in ₤.

Consider, for example, the occurrence of 'denotes' in C, represented by 'denotes_{cC}'. The c_C-schema can evaluate all denoting expressions of ₤, except for C and any other singularities of 'denotes_{cC}'. But these singularities can be evaluated by other reflective schemas. As we saw in Chapter 2, we can go on to rehabilitate C – when evaluated by the reflective c_R-schema, for example, C denotes (that is, denotes_{cR}) is π+6. Moreover, the phrase 'the number denoted by C' produced in this reflective context, represented by 'the number denoted_{cR} by C', can be evaluated by the c_C-schema without contradiction. Moving on to the theoretical languages $\mathfrak{T}$ and $\mathfrak{T}$*, none of their denoting expressions are identified as singularities of 'denotes_{cC}', and they can all be evaluated by the c_C-schema without contradiction. So all 'decontextualized' denoting expressions, such as K and K^+, are in the extension of the occurrence of 'denotes' in C, and any other occurrence of 'denotes'.

The threat of revenge appears in the form of König-like phrases in a decontextualized language. Phrases such as K and K^+ are special because they pick out objects that no expression of the target language ₤ can pick out. But this does not compromise the singularity solution. For an ordinary use of 'denotes', say 'denotes' in C, has the phrases K and K^+ in its extension.[26] The c_C-schema can evaluate these phrases. The only expressions that the c_C-schema cannot evaluate are the singularities of 'denotes_{cC}' – but these can be evaluated by suitably reflective schemas associated with occurrences in ₤ of 'denotes' in other contexts. No denoting phrase, whether of ₤, or an analogue of ₤, or a decontextualized, theoretical language, escapes the evaluative reach of the language ₤.

The response to revenge runs parallel in the cases of extension and truth. In the case of extension, where £ is now *L*+'extension', we can form the 1-place predicate 'predicate of £ that has an extension$_\alpha$ for some context α'. This predicate will apply not only to expressions of £ such as M ('moon of the Earth') but also to predicates of £ like P ('unit extension of a predicate on the board in room 213'), since P does have a determinate extension$_\alpha$ for a suitably reflective context α. And we can form the 1-place predicate 'extension$_\alpha$, for some context α, of a predicate of £', or 'extension$_{\pounds}$' for short. Now we can form a decontextualized Russell predicate R associated with revenge: 'non-self-membered extension$_\alpha$, for some context α, of a predicate of £', or 'non-self-membered extension$_{\pounds}$' for short. The extension E_R of this predicate cannot be the extension$_\alpha$ of any predicate token of £, for any α. For suppose E_R is the extension$_\beta$ of a token Q of £, for some β -- that is, suppose that E_R is an extension$_{\pounds}$. Then E_R is a self-membered extension$_{\pounds}$ if and only if it is a non-self-membered extension$_{\pounds}$. If we add 'extension$_{\pounds}$' to $\mathfrak{I}$ to obtain $\mathfrak{I}^*$, then R is an expression of $\mathfrak{I}^*$ with a well-determined extension E_R shared by no expression of £. But, according to the singularity theory, E_R is in the extension of any occurrence of 'extension' in £ -- the decontextualized Russell predicate R is not a singularity of any occurrence of 'extension' in £.

As in the case of denotation, we can introduce a broader extension operator which applies not only to £ but also to analogous fragments of natural languages other than English. And then we can introduce a further Russell predicate R^+: 'non-self-membered extension$_\alpha$, for some context α, of a predicate of £ or any analogue of £.' The extension of R^+ is shared by no expression of £ or its analogues – but R^+, like R, is in the extension of any occurrence of 'extension' in £. Also in parallel with the case of denotation, the language $\mathfrak{I}^*$ cannot contain the predicate 'extension of a predicate of $\mathfrak{I}^*$', on pain of Russell's paradox tailored to £. $\mathfrak{I}^*$ is at the base of a hierarchy of classical, decontextualized languages, where each subsequent language contains the extension operator for the previous one. All of the predicates of these languages are in the scope of any occurrence of ‘extension’ in ₤, since none are identified as singularities.

As in the case of denotation, the expressive power of ₤ is sufficient for the evaluation of any predicate – whether a predicate of ₤, or an analogue of ₤, or $\mathfrak{I}$ or $\mathfrak{I}^*$, or a language of a hierarchy generated from $\mathfrak{I}^*$. The scope of an occurrence of ‘extension’ in ₤ is global except for its singularities – and the singularities beyond the scope of one occurrence of ‘extension’ is within the scope of another in a suitably reflective context.

Similarly in the case of truth. We obtain $\mathfrak{I}$* by adding to $\mathfrak{I}$ the predicate ’true$_{₤}$’, i.e. ‘is a sentence of ₤ that is true$_{\alpha}$ for some context α’. This predicate may be regarded as the truth predicate for ₤, since it applies to exactly the sentences of ₤ that are true in some context – including pathological sentences such as the liar sentence L (‘The sentence written on the board in room 213 is not true.’) Like ‘denotes$_{₤}$’ and ‘extension$_{₤}$’, the extension of ’true$_{₤}$’ is not shared by any predicate of ₤, since every occurrence of ‘true’ in ₤ has singularities that are true when evaluated by suitably reflective schemas. We can now form this liar-like sentence associated with revenge:

(X) X is not true$_{₤}$.

X says that X is not a sentence of ₤ that is true$_{\alpha}$ for some context α. Suppose X is a sentence of ₤ that is true in some context, say β. Then X is not true in any context, including β. We reach a contradiction -- so X is a true sentence. That is, it’s a true sentence of $\mathfrak{I}$*, a true sentence that can’t be expressed in the language ₤. This is what is special about the revenge-related sentence X: it is a true sentence that cannot be expressed in ₤. But the singularity theory accommodates

X. X is a true, decontextualized sentence of ℑ*, and since it is not identified as a singularity, it is in the extension of any occurrence of 'true' in ₤.

X is also in the extension of 'true-in-ℑ*', the truth predicate for ℑ* contained not in ℑ*, on pain of contradiction, but in a metalanguage for ℑ*. Again, a hierarchy of classical, context-independent languages is generated from ℑ*, each containing the truth predicate for the previous language. No context-independent language contains a global truth predicate – we can always 'diagonalize out'. If the hierarchy is cumulative, a language at a given level will have a truth predicate with scope over all the truths of ₤ and the truths of the languages at lower levels. But it will not have scope over the truths of the language that contains it, or of languages higher in the hierarchy. But the language ₤ -- a portion of the natural language English -- can accommodate this hierarchy: according to the singularity theory, every true sentence of the hierarchy is in the extension of any occurrence of 'true' in ₤.

No sentence escapes the evaluative reach of the language ₤, just as no denoting expression or predicate escapes its reach. An occurrence of 'true' in ₤ applies everywhere except to its singularities – apart from its singularities, it applies to the truths of ₤, the truths of any analogue of ₤, and to the truths of context-independent languages such as ℑ and ℑ* and those of the hierarchy generated from ℑ*. Our omniscient being, as it writes a liar sentence on the board in room 213, knows all about these truths, and can place them in the extension of the token of 'true' on the board. This token will have singularities -- in particular the sentence on the board. But these singularities are also in the evaluative scope of ₤ -- they are in the extension of other tokens of 'true' that occur in suitably reflective contexts.[27]

It's worth stressing that the threat of revenge is handled here by a familiar distinction between languages that contain context-sensitive expressions and those that don't. No new semantic concepts or special machinery is needed. In particular, revenge is handled by a distinction between languages that contains context-sensitive expressions (‘denotes’, ‘extension’, ‘true’), and classical, theoretical languages that contain no context-sensitive expressions. Revenge-related expressions show that there are things that can be said in a context-independent language that cannot be said in ₤. But these revenge expressions are nevertheless handled by ₤ -- these expressions are in the extension of every occurrence in ₤ of ‘denotes’ or ‘extension’ or ‘true’.

At this point, it may also be worth emphasizing again that the singularity approach does not offer a stratified treatment of our ordinary semantic terms: 'denotes', 'extension' and ‘true’ are not stratified into a series of increasingly comprehensive predicates. Instead we have in each case a single, context sensitive term, and any occurrence of the term has singularities that other occurrences do not have. No occurrence is more (or less) comprehensive than another; each occurrence is minimally restricted. This feature of the singularity account shouldn’t be obscured by the hierarchy generated from $\mathfrak{I}^*$, any more than by the reflective hierarchy developed in Chapter 6.[28] The hierarchy generated from $\mathfrak{I}$* are generated from classical theoretical languages, and a parallel hierarchy could be generated from, say, the language of arithmetic or chemistry, or any suitably regimented language. In each case we will obtain an infinite series of denotation (extension, truth) predicates - one series starting with, for example, the predicate 'denotes-in-$\mathfrak{I}$*’, another with the predicate 'denoting expression in the language of arithmetic', and so on. But these series are composed of predicates of the form 'denotes-in-Q', limited to some suitable scientific language or metalanguage Q. In contrast, our interest lies in the English predicates 'denotes', ‘extension’ and ‘true’ - and according to the singularity solution, this is a context-sensitive predicate that applies to denoting expressions at every level of all of these hierarchies.

We’re interested in the semantic predicates of *natural* languages, and we want them to share the intuitively unconfined expressive power of natural language.

So we do not take the formal hierarchy generated from, say, $\mathfrak{I}^*$ to explicate our ordinary concepts. The levels do not correspond to any stratification of the context-sensitive predicates 'denotes', ‘extension’ or ‘true’. The singularity proposal abandons this Tarskian route. For the Tarskian, questions about the extent of the hierarchy and quantification over the levels will present special difficulties. Of course, these are substantial questions, quite independently of any particular proposal about the semantic expressions of English. But they present no *special* difficulty for the singularity account. According to the singularity account, an ordinary context-sensitive use of 'denotes', or ‘extension’ or ‘true’ applies almost everywhere, failing to apply only to those expressions that are singularities. By Minimality, any use of 'denotes' captures in its scope any denoting phrase that is not a singularity of that use – and that includes denoting expressions from every level of the hierarchies generated from $\mathfrak{I}$* and $\mathfrak{I}^+$, whatever the extent of these hierarchies.

There are of course legitimate questions about these hierarchies. What is their extent? Can we quantify over all their levels? Must we resort to schematic generalizations? If we were offering a Tarskian account of revenge, these questions would be critical. But we are offering a different kind of contextual account, and so these questions are less urgent (though no less interesting). They are questions that do not bear directly on the singularity theory, because the theory does not stratify ‘true’, ‘denotes’ or ‘extension’. These semantic terms of ordinary English apply to the language of the theory, and beyond. Moreover, the context-independent expressions of these hierarchies play no role in determining the values of pathological expressions containing ‘denotes’, ‘extension’ and ‘true’. Determining those values is a matter of identifying singularities – and the determination trees of Chapter 6 frame this procedure.

If we mean by a semantically universal language a language with unrestricted, fully global semantic predicates, then the paradoxes show us that we have to give up on semantic universality. No occurrence of a semantic predicate of ₤ is completely unrestricted in its scope (each has singularities). And no semantic predicate of a context-independent language is global (we can always diagonalize out). But if by a semantically universal language we mean a language where every expression can be semantically evaluated, then we can respect the intuition that natural languages, like English, are semantically universal. Any use of a semantic predicate of ₤ -- a portion of English -- is as close to universal as possible. The scope of these semantic predicates reflect the expressive power of natural language. And what a given use of ‘denotes’, ‘extension’, or ‘true’ cannot reach will be reached by another use of that term in another (reflective) context.

9.4 Summary

This is perhaps a good place to sum up some of the interconnected aims of the singularity theory:

(i) Provide a unified account of the semantic paradoxes.

(ii) Respect our ordinary, intuitive reasoning in the presence of paradox. Do not leave natural language behind.

(iii) Minimize any revision to our semantic concepts.

(iv) Respond adequately to revenge.

(v) Respect as far as possible the intuition that natural languages are semantically universal.

(vi) Preserve classical logic and semantics.

In regard to (i), the singularity theory encompasses denotation, extension, truth, and truth-of. In regard to (ii), the theory respects *repetition* and *rehabilitation* as sound reasoning, to be explained in contextual terms. The reasoning is not artificially blocked; and our semantic concepts are not replaced by artificial concepts unfamiliar to the ordinary speaker. In line with (iii), the theory places minimal restrictions on our uses of semantic terms. And, as we've seen in this chapter, (iii) provides the key to (iv) and (v). A semantic term applies everywhere except to its singularities. So the term's application extends to expressions of the singularity theory itself, and indeed to expressions of any decontextualized or context-independent language. And those singularities that are beyond the reach of a given use of one of our ordinary semantic terms can be reached by uses of those terms in suitably reflective contexts.

Finally, the singularity theory preserves classical logic and semantics. We saw in Chapter 2 that the contextual analysis of repetition, rehabilitation and iteration treats the reasoning as classically valid. Further, the principle of bivalence is upheld, even for pathological sentences. Consider the liar sentence L: L is neither $true_{cL}$ nor $false_{cL}$, but this truth value gap is closed upon reflection: L is true, when reflectively evaluated. And L is $true_{£}$, since it is true in some context. A Truth-Teller sentence – say, (T) T is $true_{cT}$, from Chapter 5 -- is false on reflection, and is $false_{£}$. In general, all the sentences of £, even the pathological ones, are either $true_{£}$ or $false_{£}$. And the law of excluded middle holds, even for pathological sentences. For instance, Lv~L holds, since the first disjunct holds (L is indeed not $true_{cL}$); Tv~T holds, since it is not the case that T is $true_{cT}$.

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Notes to Chapter 9

1. See Parsons 1974a, Burge 1979, Barwise and Etchemendy 1987, Gaifman 1988 and 1992, Glanzberg 2001. For fuller accounts of these contextual theories than I provide here, see Simmons, forthcoming. The contextual-hierarchical account is endorsed in Koons 1992. In Chapter 6 of Koons 1992, Koons argues that the hierarchical theories of Burge 1979, Barwise and Etchemendy 1987, and Gaifman 1988, 1992 are special cases of a more general theory, and then, in Chapter 7, applies this theory to doxic paradoxes. For further discussion of Koons, see, for example, Juhl 1997.

2. Burge 1979.

3. In his presentation of the formal theory, Burge explicitly accommodates only finite levels. As he acknowledges in the postscript to Burge 1979, provisions would need to be made for extending the constructions into the transfinite (see Burge, in Martin 1984, p.115).

4. According to Burge, the level of a sentence is established in context by certain pragmatic principles of interpretation. See Burge 1979 and 1982.

5. Parsons 1974a, p.35.

6. *ibid*.

7. For a full presentation of the reasoning here, see Glanzberg 2001, p.229.

8. The challenge now is to show how this expansion fits with standard ideas from linguistics and philosophy of language. To meet this challenge, Glanzberg makes use of the idea that context provides a running record of information available at a given point in a discourse. In particular, Glanzberg draws on the extensive work in the literature on the notions of *salience* and *topic*, in order to make it clear that taking context to include a running list of salient items is well-motivated, and quite independent of the liar.

9. For situation semantics, see Barwise and Perry 1983.

10. See Aczel 1987.

11. For further discussion of Barwise and Etchemendy's theory, see, for example, Grim and Mar 1989, McGee 1991, Koons 1992, Glanzberg 2004, Gauker 2006.

12. One might argue: either the sentence is true at some level, say i, or it is not true at any level. If the former, then, given what the sentence says, the sentence is not true at any level, including i – and we reach a contradiction. If the latter, then the sentence is true – and since truth is always tied to a level, it is true at some level. And we have a contradiction again.

13. See Burge 1979, p.108.

14. On this feature of indexicals such as ‘I’, ‘here’, ‘you’, ‘now’, see, for example, Stanley 2000.

15. Burge discusses schematic uses of ‘true’ in Burge 1979, in Martin 1984, pp.115-7, and pp.107-8, and in Burge 1982, p.353.

16. Burge, in Martin 1984, p.116.

17. See Williamson 2003.

18. Cf. Russell 1908, in van Heijenoort 1967, pp.156-159. Relatedly, Parsons considers the following objection to his account in Parsons 1974a: if we interpret the quantifiers of Parsons' paper as ranging over some sufficiently large set, we can then produce a discourse to which his analysis of the liar will not apply. In response, Parsons suggests that the generality that his paper has, transcending any particular set as the range of the quantifiers, “must lie in a sort of systematic ambiguity, in that indefinitely many such sets will so” (Parsons 1974a, fn.13, p.28). The notion of typical ambiguity is also related to Burge's claim that there are schematic uses of ‘true’.

19. For critical discussion of the claim that there is no absolutely unrestricted quantification, see Williamson 2003 and the papers in Rayo and Uzquiano 2006.

20. See Barwise and Etchemendy 1987, Chapter 13.

21. See, for example, Kripke in Martin 1984, p.80 and fn.34, and for further discussion, Simmons 1993, Chapter 3.

22. Francois Recanati 2002, p.312. There may well be differences between predicates like ‘small’ which trigger a search for a comparison class, and those like ‘hexagonal’ or ‘flat’ that don't. But such differences don't bear on the present point.

23. Jason Stanley has taken contextualists about truth to be claiming that ‘true’ is a narrow indexical, and he argues that this claim is put into doubt by the observation that the vast number of cases of unobvious context dependence do not involve narrow indexicality (Stanley 2000, pp.430-1). However, the singularity theory does not take ‘true’ to be a narrow indexical, but an instance of the “vast number of cases” of unobvious context dependence. Stanley's objection does apply to Burge – but on Burge's behalf, one might wonder why, given that ‘true’ presents the special case of a deep paradox, the unobvious context dependence of ‘true’ should go the way of other cases of unobvious context dependence.

24. The denotation of this phrase will depend on cardinality considerations - in particular, the cardinality of the set of contexts, and the size of the vocabulary of £.

25. We see here specific reasons why T^* cannot be a metalanguage for ₤ - but in general there are reasons why we should not expect any theory of a context-sensitive term to be a Tarskian metalanguage. Such a theoretical language will be 'austere', free of indexical terms, 'scientific'. The language in which we give an adequate semantics for 'I' or 'now' or ‘small’ will not itself contain the context-sensitive terms 'I' or 'now' or ‘small’. There is no requirement that a theory of 'I' or 'now' provide context-sensitive means for referring to myself or the present time. So we should not expect an utterance of 'I am hungry' or 'The meeting starts now' to be translatable into the language of the theory. In general, we should not expect that there will be a way of translating the context-sensitive term into a term or phrase of the language of the theory. Now a Tarskian metalanguage "must contain the object language as a part", or at least it must be the case that "the object-language can be translated into the metalanguage" (Tarski 1944, in Blackburn and Simmons 1999, p.126). So in general a language in which we state the theory of a context-sensitive term will not be a Tarskian metalanguage. This point in stressed by Kaplan, in Kaplan 1997, p.7.

26. It is one thing for there to be an expression in another language that has a denotation that no expression of ₤ has. It is another for there to be an expression of another language that is not in the extension of an occurrence of ‘denotes’. For an everyday example, a French denoting phrase is not a phrase of ₤ (which is a fragment of English). But the French phrase is not excluded from the extension of ‘denotes’. Similarly, K and K^+ have denotations that no expression of ₤ has, but both are in the extension of any occurrence of ‘denotes’.

27. It is straightforward to provide a parallel account of the predicate ‘true of’. A revenge-related predicate here will be ‘predicate of ₤ that is not true_α of itself, for some context α’.

28. As we saw in Chapter 6, the reflective hierarchy is a convenient way to organize the expressions of ₤ that contain the semantic expression *t*: we regiment these expressions according to the highest level of reflection that they involve. But as we said in Chapter 6, the level of an expression in the reflective hierarchy is not at all a measure of the extension of the occurrence of *t* in the expression. The reflective hierarchy gives formal expression to our capacity to reflect on pathological expressions - but the hierarchy does not stratify *t*. And neither do the hierarchies generated from T^* or T^+.