

## Chapter 3

### Singularities

#### 3.1 Minimality and singularities

Recall our simple paradox of denotation, and the contextual treatment of *repetition*. At the second stage, C is evaluated by the unreflective  $c_C$ -schema; at the fourth stage, the repetition  $C^*$  is evaluated by the reflective  $c_E$ -schema. We saw that C (and  $C^*$ ) are not in the extension of 'denotes $_{c_C}$ ', and  $C^*$  (and C) are in the extension of 'denotes $_{c_E}$ '.<sup>1</sup> So C and  $C^*$  are excluded from the extension of 'denotes $_{c_C}$ '. We can ask: what else is excluded from the extension of 'denotes $_{c_C}$ '? And what is the relation between the extensions of 'denotes $_{c_C}$ ' and 'denotes $_{c_E}$ '? We can ask parallel questions about the expressions 'extension $_{c_P}$ ' and 'extension $_{c_E}$ ', and 'true $_{c_L}$ ' and 'true $_{c_\Sigma}$ '.

A possible response here is a Tarskian one: the expressions in each pair are associated with distinct levels of language.<sup>2</sup> For example, the predicate 'denotes $_{c_C}$ ' is the denotation predicate of the language associated with C's unreflective context of utterance; the predicate 'denotes $_{c_E}$ ' is the more comprehensive denotation predicate of a semantically richer language associated with a context reflective with respect to C. On such a hierarchical account, the extension of 'denotes $_{c_C}$ ' is properly contained in the extension of 'denotes $_{c_E}$ '. Similarly for 'extension $_{c_P}$ ' and 'extension $_{c_E}$ ', and 'true $_{c_L}$ ' and 'true $_{c_\Sigma}$ '.

Some kind of hierarchical resolution is perhaps the orthodoxy regarding the semantic paradoxes.<sup>3</sup> But an immediate worry about the hierarchical approach is that it offers too regimented an account of natural language. Surely English does not contain infinitely many

distinct denotation predicates, but just one – and similarly for extension and truth.<sup>4</sup> And surely the stratification of English into a hierarchy of distinct languages is unnatural. As we saw in Chapter 1, Tarski himself had doubts about such a treatment of natural language. And Russell once described the appeal to hierarchy as “harsh and highly artificial”.<sup>5</sup>

Further, it is hard to see how levels can be assigned to occurrences of the denotation predicate in any systematic way.<sup>6</sup> How are we to interpret a given phrase containing the predicate 'denotes'? To which level of language does it belong? Except in very simple cases, it seems that we will have little basis for an assignment of one level rather than another. And what level should we assign to a global statement like 'Every expression of English either denotes a number or does not denote a number'? Any assignment of a level here will compromise the global nature of the statement.

But perhaps the most serious worry about the hierarchical approach is that it places massive restrictions on ordinary uses of our semantical terms. Consider for example a Tarskian stratification of 'denotes'. On a standard hierarchical line, the expression 'the only even prime' is of level 0; the expression "the number denoted by 'the only even prime'" is of level 1, and so on, through the levels. Your use of 'denotes' in an utterance of level 1 has in its extension all referring expressions of level 0, *and no others*. So all sentences of level 1 and beyond are excluded from the extension of such a use of 'denotes'. Gödel remarks of Russell's simple theory of types that

“objects are divided into mutually exclusive ranges of significance, each range consisting of those objects that can replace each other; and that therefore each concept is significant only for arguments belonging to one of those ranges, i.e., for an infinitely small portion of all objects.”<sup>7</sup>

A similar complaint can be made about a standard hierarchical account of denotation: ordinary uses of 'denotes' will apply to just a fraction of all the expressions that denote. Similarly with 'extension' and 'true'.

The account I shall offer is in a strong sense *anti-hierarchical* – there is no stratification of 'denotes', 'extension' or 'true'. The leading idea is that semantic pathology forces only *minimal* restrictions on occurrences of these expressions. We can call the guiding principle here *Minimality*: the application of these semantic expressions is to be restricted only when there is reason to do so. By adhering to Minimality, we respect a basic semantic intuition about predicates. Intuitively, we take a predicate to apply to everything with the property that the predicate picks out (or, if the predicate is 2-place, all pairs standing in the relation that the predicate picks out, and so on for many-place predicates). If an individual has the property ascribed by the predicate  $\varphi$ , then that individual is in the extension of  $\varphi$ . The more restrictions we place on occurrences of our semantic expressions, the more we are at odds with this intuition. We do expect any solution to a genuine paradox to require some revision of our intuitions. But the more a solution conflicts with our intuitions, the less plausible the solution will be.

For example, suppose you say "The square of 1' denotes 1". Here, your use of 'denotes' is quite neutral and unproblematic. Should the pathological token C be excluded from its extension? (More precisely, should the pair  $\langle C, \pi+6 \rangle$  be excluded from its extension?) Minimality says no -- because there is no need to exclude it. We have seen that C can be *rehabilitated*. When you re-evaluate C in the reflective context  $c_R$ , you find that C denotes (that is,  $\text{denotes}_{c_R}$ )  $\pi+6$ . This is because the sum of the numbers  $\text{denoted}_{c_C}$  by expressions on the board is *indeed*  $\pi+6$ . It's a fixed semantic fact that the sum of the numbers  $\text{denoted}_{c_C}$  by expressions on the board is  $\pi+6$ . And so C can be counted as a denoting expression in your

context of use, just as it can be so counted in an explicitly reflective context. We have no reason to suppose that in your context of use, C must be evaluated by the contradiction-producing  $c_C$ -schema -- your context of use is *neutral* with respect to C.<sup>8</sup> And, by Minimality, if we can count C as a successful denoting expression in your context of use, then we must so count it. Just as the fourth and final stage of *rehabilitation* places C in the extension of ‘denotes<sub>cR</sub>’, so Minimality requires that C be placed in the extension of your neutral use of ‘denotes’.

Minimality keeps surprise to a minimum: each of our uses of 'denotes' applies to almost all denoting phrases, each of our uses of ‘extension’ applies to almost all expressions that have extensions, and each of our uses of ‘true’ apply to almost all the truths. We are sometimes forced to restrict the scope of a semantic expression -- we must, for example, limit the extension of the token of 'denotes' in C by excluding C itself. Still, according to Minimality, we exclude only those denoting expressions that cannot be included. So my proposal identifies what I shall call *singularities* of denotation, extension and truth. If, for a context  $\alpha$ , a denoting phrase  $\sigma$  cannot be assessed by the  $\alpha$ -schema, then  $\sigma$  is a *singularity* of 'denotes <sub>$\alpha$</sub> '. And if  $\sigma$  is a singularity of 'denotes <sub>$\alpha$</sub> ', and the  $\alpha$ -schema is  $\sigma$ 's evaluating schema, then  $\sigma$  is *pathological*. So C is a singularity of 'denotes<sub>cC</sub>', and it is pathological too, since, when C is evaluated, at the second stage of *repetition*, C's evaluating schema is the  $c_C$ -schema.<sup>9</sup> C\* is also a singularity of 'denotes<sub>cC</sub>', but C\* is *not* pathological, since, when C\* is evaluated, at the fourth stage of *repetition*, its evaluating schema is the  $c_E$ -schema, and C\* does denote<sub>cE</sub> a number. Similarly, P and P\* are singularities of ‘extension<sub>cP</sub>’, and P is pathological, but P\* is not. And L and L\* are singularities of ‘true<sub>cL</sub>’, and L is pathological, but L\* is not.

According to Minimality, then, C is not a singularity of the occurrence of ‘denotes’ as it occurs in your utterance "‘The square of 1’ denotes 1". What exactly is it about your context of



C repeats on an infinite branch, indicating that it's pathological. But N stands above the loop in which C is caught. N's context stands apart from C's pathology – N isn't written on the board and isn't looped with itself, or with C.

In both these cases, your utterance is outside the pathological circle in which C is caught. Suppose that, in either case, you were apprised of C's pathology. Then you could, within your neutral context, reason through C's pathology, and reflectively establish a value for C. (As we've just seen, the value for C will be  $\pi+6$ , since the sum of the numbers denoted<sub>cC</sub> by expressions on the board in room 213 is indeed  $\pi+6$ .) You would have no reason to treat C as a singularity of your use of 'denotes'. This is what Minimality recommends: treat C as a singularity only if you have to. In a nutshell, Minimality treats your context of utterance as *reflective with respect to C*.

It's the same story with P and L. If in some suitably neutral context, I talk about the extensions of the predicates on the board in room 213, then, by Minimality, my use of 'extension' will have P in its scope (since P has a reflectively established extension) -- or if I say that the sentence on the board in room 213 is true, then my use of 'true' will have L in its scope (since L has a reflectively established truth value, true). My neutral contexts of utterance are *reflective with respect to P and L*.

### 3.2 Reflective Status

In the previous chapter, I identified *reflective status* as a contextual parameter to which 'denotes', 'extension', and 'true' are sensitive. We can now say more about the reflective status of a context. I'll focus on the case of denotation, but the cases of extension and truth run parallel.

Let  $\sigma$  be an expression in which ‘denotes’ (or ‘extension’ or ‘true’) occurs, let  $c_\sigma$  be  $\sigma$ ’s context of utterance, and let  $\rho$  be a pathological denoting expression. When is  $c_\sigma$  reflective with respect to  $\rho$ ? The intuitive answer is this: when  $c_\sigma$  can accommodate  $\rho$ ’s pathology and a reflective evaluation of  $\rho$ . In the cases of *repetition* and *rehabilitation*, the accommodation is explicit – in the contexts associated with the third and fourth stages of *repetition* and *rehabilitation*, it is part of the common ground that C (or P or L) is pathological. But the accommodation need not be explicit. If  $\sigma$  stands above  $\rho$ ’s pathology, then  $c_\sigma$  is reflective with respect to  $\rho$ . And for  $\sigma$  to stand above  $\rho$ ’s pathology is a matter of the semantic network that  $\sigma$  generates. For example, the context in which you produce N is reflective with respect to C in virtue of the semantic network that N generates. In your context of use, N generates a semantic network in which it stands above the loop in which C is caught. Of course, we’ll need a more precise account of what it is for an expression to ‘stand above’ pathology – and that will require a more formal account of semantic networks. But I hope that for now the intuitive idea is clear.

When you produce N, you may not know the reflective status of your context of utterance  $c_N$ . You may not know what is written on the board at that time in room 213; you may not know the semantic network that N generates. Still, the context  $c_N$  has these fixed features that determine its reflective status with respect to C. The contextual features that determine  $c_N$ ’s reflective status, and the extension of your use of ‘denotes’ – in particular, that C is in its extension – may be independent of what you know.<sup>10</sup>

But suppose you suddenly became omniscient. In your neutral context, you now know what is written on the board. You can recognize C as pathological, reason through its pathology, and reflectively assign a value to it ( $\pi+6$ ). That is, you can go through the reasoning that constitutes *rehabilitation*. (And then you can assign a value -- ( $\pi+6+(\pi+6)$ ) -- to your own

utterance N). Your context of use can accommodate C's pathology and a reflective evaluation of C. The reflective status of  $\alpha$  is determined independently of what you know – it is N's semantic network, together with Minimality, that establishes that  $c_N$  is reflective with respect to C. However, this feature of context  $c_N$  is true to the reasoning you would carry out if you knew all the facts, and with the reasoning that we *do* carry out when C's pathology is part of the common ground. So the contextual coordinate *reflective status*, to which 'denotes' is sensitive, is suitably tied to how speakers reason with the denotation predicate in the setting of the paradoxes.

A speaker who is familiar with *repetition* and *rehabilitation* can readily project from those discourses to cases where  $\rho$ 's pathology is not part of the common ground. Suppose you are familiar with *repetition* and *rehabilitation*. Suppose you produce N without knowing what is written on the board. You will nevertheless have a general recipe for determining the value of N: *determine the denotations* (that is,  $\text{denotations}_{c_N}$ ) *of the expressions on the board, reflectively wherever appropriate*, where a reflective evaluation is appropriate whenever an expression on the board is pathological, but not pathologically tangled with N. Your familiarity with *repetition* and *rehabilitation* prepares you for three possibilities. An expression on the board might be a straightforward denoting phrase such as 'six' – here the notion of a reflective evaluation has no role to play. Or it might be pathological, where N stands above its pathology. Here you would reflectively establish a value for it, and proceed from there. Or it might be pathologically tangled with N, in which case you would find N to be pathological too. As the second and third cases make clear, the general recipe you follow treats your use of 'denotes' as sensitive to the reflective status of your context with respect to each of the expressions on the board. The general recipe you draw from *repetition* and *rehabilitation* identifies *reflective status* as the contextual coordinate to which 'denotes' is sensitive – even though you don't know the specifics

of the semantic network generated by N in its context of use. In the case where A, B and C are written on the board, this general recipe will have the specific result that  $c_N$  is reflective with respect to C (though you won't know that this is a specific result of your general recipe).

You'll also be able to recognize that if there is a pathological expression  $\rho$  on the board that requires a reflective evaluation, then there is a crucial difference between  $\rho$ 's context of use and yours. Your context of use is reflective with respect to  $\rho$ , and  $\rho$  will not be a singularity of your use of 'denotes'. In contrast,  $\rho$ 's context of use is not reflective with respect to  $\rho$ , and  $\rho$  will be a singularity of the occurrence of 'denotes' in  $\rho$ . There will be a difference in the extensions of these occurrences of 'denotes', a difference tied to the reflective status of their respective contexts of use. So even if you are ignorant of  $c_N$ 's specific reflective status with respect to this or that pathological expression, you have a general appreciation of the role that the reflective status of a context plays in determining the extension of a use of 'denotes'.<sup>11</sup>

There are, then, two ways in which a context can be reflective with respect to a pathological expression  $\rho$ . Take again an expression  $\sigma$  in which 'denotes' -- or 'extension' or 'true' -- occurs, where  $c_\sigma$  is its context of use. First, the context  $c_\sigma$  can be *explicitly reflective* with respect to  $\rho$  -- it can be part of the common ground in  $c_\sigma$  that  $\rho$  is pathological. Second, the context  $c_\sigma$  can be *non-explicitly reflective* with respect to  $\rho$  -- the semantic network generated by  $\sigma$  shows that  $\sigma$  stands above  $\rho$ 's pathology.

Given these two ways in which  $c_\sigma$  can be reflective with respect to  $\rho$ , there is only one way in which  $c_\sigma$  can *fail* to be reflective with respect to  $\rho$ : when  $\sigma$  is pathologically tangled with  $\rho$ . The details of  $\sigma$ 's semantic network may well not be known to the speaker -- for example, I may write C on the board while confused about my whereabouts. But, if I'm aware of how we use 'denotes' in *repetition* and *rehabilitation*, I will have a general procedure for determining a

value for C. As before, the procedure is this: *determine the denotations of the expressions on the board, reflectively wherever appropriate*. As we saw, this general procedure allows for the eventuality that a reflective evaluation may not be immediately possible – what I’ve written may not stand above pathology, and its context may fail to be reflective. This is the case with C, in contrast with N. Again, the contextual role that reflective status plays can be appreciated by speakers who do not know the specific reflective status of their contexts of use.

It’s evident that a context can be non-explicitly reflective without being explicitly reflective – the context of use of your utterance N is a case in point. In the other direction, a context can be explicitly reflective without being non-explicitly reflective. For example, this is so in any case of a repetition. C and C\* determine exactly similar semantic networks, but while C\*’s context is explicitly reflective with respect to C, C’s context is not.<sup>12</sup> A context can explicitly fail to be reflective – for example, when we intentionally produce a pathological phrase. And it’s evident that a context can non-explicitly fail to be reflective – for example, when I unwittingly write C on the board.

C is a singularity only in a context-relative way -- there are reflective contexts in which C *is* in the extension of 'denotes'. C is a singularity of only those occurrences of 'denotes' in contexts that are not reflective, either explicitly or non-explicitly, with respect to C.<sup>13</sup> We will see later in a more formal way that there is a tight connection between reflective contexts and singularities. Call a context *reflective with respect to an expression* if it is either explicitly or non-explicitly reflective with respect to that expression. Let  $\alpha$  be a context,  $\rho$  a pathological expression, and  $t$  a semantic term (either ‘denotes’, ‘extension’ or ‘true’). Then  $\rho$  is a singularity of ‘ $t_\alpha$ ’ if and only if  $\alpha$  is not reflective with respect to  $\rho$ , where ‘ $t_\alpha$ ’ represents a use of the term  $t$  in context  $\alpha$ . Equivalently,  $\alpha$  is reflective with respect to  $\rho$  if and only if  $\rho$  is not a singularity of

‘ $t_\alpha$ ’. This latter biconditional provides for a characterization of a reflective context. Of course, this characterization requires an independent account of the notions of *pathology* and *singularity*, and that will be the central task of the formal theory.

As will also become clearer, I assume very little about the nature of contexts, about what a context of use is. I do assume that explicit reflective status is a determinate feature of a context; that is, I assume it is a determinate matter whether, for a context  $\alpha$  and a pathological expression  $\rho$ , it is part of the common ground in  $\alpha$  that  $\rho$  is pathological. And I assume that non-explicit reflective status (with respect to a given pathological expression) is also a determinate feature of a context – a feature fixed by semantic interconnections between expressions.

We noted in the previous chapter that ‘I’ triggers a search for the appropriate speaker, ‘small’ triggers a search for a relevant comparison class, and that ‘hexagonal’ and ‘flat’ trigger searches for the standards of precision supplied by the context. Once we’re aware of the paradoxes of denotation, extension and truth, and once we’re familiar with *repetition* and *rehabilitation*, we’ll have available the general recipe above for evaluating an expression  $\sigma$  containing ‘denotes’ or ‘extension’ or ‘true’. This recipe indicates that these semantic terms trigger a search for the standards of assessment supplied by  $\sigma$ ’s context  $c_\sigma$ . Those standards will be a matter of which pathological expressions can be assessed in  $c_\sigma$  in the light of their pathologicality. And that in turn is a matter of either the information immediately available in  $c_\sigma$  (the common ground), or the semantic network generated by  $\sigma$ , or both. In short, ‘denotes’, ‘extension’ and ‘true’ trigger a search for reflective status. Even if we don’t complete the search (we may not know the empirical facts about where and when expressions are written or uttered), we know how the search is to be conducted.

### 3.3 More on singularities

No occurrence of 'denotes' or 'extension' or 'true' is without singularities. Take any innocent use of 'denotes' – for example, suppose you say: “The number denoted by ‘pi’”. You could now append the following words: “plus the number denoted by ‘six’, plus the sum of the numbers denoted by phrases in this utterance”. There is no shift to a reflective context within your expanded utterance, and so the three occurrences of the denotation predicate have the same extension. It’s easy to check that the last eleven words of your expanded utterance form a pathological denoting phrase – and this phrase is a singularity of the occurrences of ‘denotes’ in your utterance, and, in particular, of your initial innocent use. Or suppose you call attention to some extension – suppose you say: “the extension of ‘moon of the Earth’”. Suppose you add the following words, and you name your expanded utterance U: “and an empty extension of an eight-word predicate in U”. The predicate token composed of the last eight words of U is a singularity of your initial use of ‘extension’.<sup>14</sup> Or if you say “‘Snow is white’ is true”, you could produce a singularity of your unexceptional use of ‘true’ by adding “but this very sentence isn’t”.

According to the contextual analysis of *repetition* and *rehabilitation* in Chapter 2, C, P and L are respectively singularities of ‘denotes<sub>cC</sub>’, ‘extension<sub>cP</sub>’ and ‘true<sub>cL</sub>’ – but they are not singularities of occurrences of ‘denotes’, ‘extension’ and ‘true’ at the reflective fourth stage of the *repetition* reasoning, where these occurrences are represented by ‘denotes<sub>cE</sub>’, ‘extension<sub>cE</sub>’ and ‘true<sub>cΣ</sub>’. But ‘denotes<sub>cE</sub>’, ‘extension<sub>cE</sub>’ and ‘true<sub>cΣ</sub>’ have singularities of their own. For example, consider your conclusion about the repetition C\*, represented by

“C\* denotes<sub>cE</sub> a number, namely  $\pi+6$ .”

Along the lines suggested in the previous paragraph, you could continue:

“And so the number denoted by C\*, plus the number denoted by 'six', plus the sum of the numbers denoted by phrases in this sentence, is irrational.”

There is no change in reflective status, and so the occurrences of 'denotes' in your continuation will also be represented by 'denotes<sub>cE</sub>'. In your continuation, the token of the type ‘the sum of the numbers denoted by phrases in this sentence’ -- call this token C<sup>+</sup> -- is pathological. The c<sub>E</sub>-schema cannot assess C<sup>+</sup>, and C<sup>+</sup> is a singularity of 'denotes<sub>cE</sub>'. Similarly, we can construct singularities of ‘extension<sub>cE</sub>’ and ‘true<sub>cΣ</sub>’.

So every occurrence of our semantic terms will have singularities. It may well be that there are no *actual* phrases uttered that force restrictions on a given occurrence of 'denotes', ‘extension’ or ‘true’; there may be no *actual* singularities. But there are always *possible* continuations that yield singularities of the given occurrence of 'denotes'.

There are any number of these possible continuations, and so any number of associated singularities – so cardinality considerations might lead one to question the appropriateness of the term ‘singularity’. But notice that, in the case of C, there is only one singularity, C itself, that is relevant to the rehabilitation of C. As we saw in Chapter 2, we rehabilitate C by concluding

(R) So the phrase C denotes<sub>cR</sub>  $\pi+6$ .

And this reflective evaluation of C depends on the identification of the singularity C, and its exclusion from the extension of ‘denotes<sub>cC</sub>’. C is an example of what in the formal theory will be called the *key singularities* -- singularities whose exclusion determines a denotation for a pathological expression. And, as we will see, in the vast majority of typical paradoxical cases,

there will be a very small number of key singularities -- often just one, as in the case of C, and the cases of P and L as well.

The example of  $C^+$  (and parallel examples that can be constructed for 'extension' and 'true') bring out the anti-hierarchical nature of the singularity proposal. Observe that we can reflect on the pathological token  $C^+$ , just as we earlier reflected on C. In a suitably reflective context, we can conclude that  $C^+$  denotes  $(\pi+6)+6$  -- since the only denoting phrases in your continuation that denote<sub>cE</sub> numbers are the first two, and they denote<sub>cE</sub>  $\pi+6$  and 6. By Minimality,  $C^+$  will have this denotation when assessed by schemas other than the cE-schema. In particular, the token *does* denote<sub>cC</sub> - it is *not* a singularity of 'denotes<sub>cC</sub>'. The cC-schema does determine a denotation for it. The relevant instance of the cC-schema may be represented as follows:

$C^+$  denotes<sub>cC</sub>  $(\pi+6)+6$  iff the sum of the numbers denoted<sub>cE</sub> by phrases in the utterance (PC) is  $(\pi+6)+6$ .

The right hand side is true, and so we may infer the left hand side.

On a hierarchical account, the extension of 'denotes<sub>cC</sub>' will be a proper subset of the extension of 'denotes<sub>cE</sub>' - but according to the singularity proposal, *neither extension includes the other*. Notice also that there is a member of the extension of 'denotes<sub>cC</sub>' in which the predicate 'denotes<sub>cE</sub>' appears. For the Tarskian, this would amount to an unacceptable mixing of language levels. On the singularity proposal, there are no such levels. As a consequence of Minimality, the singularity proposal is not hierarchical.

It seems to me an unwarranted Tarskian presumption that in the course of *repetition* or *rehabilitation*, we move to an essentially richer language. Perhaps the Tarskian view is encouraged by consideration of the utterer's epistemic situation. In the context of use in which I

unwittingly produce a pathological expression, I do not know or believe that C or P or L is pathological. I may come to know or believe this, by transcending my initial epistemic situation. But as we have already said, the extension of a semantic term is not determined by what speakers know.

Further, the Tarskian cannot help herself to this epistemic justification of the levels. For it would be quite possible for someone to produce intentionally a pathological token on the board, in full knowledge that she will go on to reflect on that utterance, *qua* pathological. We may be quite self-conscious about the production of pathological expressions; this is so, for example, when we discuss semantic paradoxes. We may produce pathological expressions in full knowledge that when assessed by a reflective schema, these tokens will receive a definite value. In this case, there is no shift in what is known, yet still the Tarskian will discern a shift in language levels.

Minimality makes room for any degree of semantic awareness. Imagine an omniscient being standing in front of the board in room 213, as I write A, B and C on the board. Our omniscient being knows that C can be rehabilitated, and that in some other suitably reflective context we can produce a successful denoting expression ‘the number denoted by C’. In the reflective context  $c_R$ , for example, we may produce an expression represented by ‘the number denoted $_{c_R}$  by C’, where that number is  $\pi+6$ . Since this expression is a successful denoting expression, the omniscient being can place it in the extension of ‘denotes’ in C. And our being can place C+ in this extension too, as the application above of the  $c_C$ -schema to C+ shows. The being knows that, though C+ is pathological and cannot be assessed by the  $c_R$ -schema, it can be rehabilitated, since the sum of the numbers denoted $_{c_E}$  by phrases in the utterance (PC) is indeed  $(\pi+6)+6$ . The reach of the occurrence of ‘denotes’ in C is almost global, and this comes close to

what we intuitively expect: ‘denotes’ should apply to *any* phrase that denotes. ‘Denotes’ is a term of English, a natural language – not a regimented or restricted formal language. Pre-theoretically, we expect ‘denotes’ to apply to any successful denoting phrase of English, or of *any* language, whatever the context in which the phrase is produced. That expectation is part of the intuition that natural languages are universal. But not even an omniscient being can place C in the extension of ‘denotes’ in C. This occurrence of ‘denotes’ applies everywhere except to its singularities.

Though Minimality is a guiding principle of the singularity account, it is not unconstrained. Suppose that today only one utterance is produced at the Great Rock:

(1)                    the number denoted by tomorrow's utterance here

$$m = \begin{cases} \\ 0, \text{ if there is no such number.} \end{cases}$$

(2)                    the number denoted by tomorrow's utterance here

$$m = \begin{cases} \\ 0, \text{ if there is no such number.} \end{cases}$$

Each day a token of the same type is produced, and so on, forever. Intuitively, the expressions (1), (2), (3), ... (n), ... form a pathological system: each heads an infinite chain. It might better accord with Minimality to treat (2), for example, as reflective. That is, we might take (3) to be pathological, because it heads an infinite chain, and then reason that since (3) does not denote a number, (2) denotes 0. And then (2) will not be excluded from the occurrence of 'denotes' in (1) - and, by Minimality, it is better to include (2) in the scope of this occurrence of 'denotes' than

exclude it. However, since (2) is related to (3) just as each sentence in the chain is related to the next, we have no reason to treat (2) differently from any other sentence in the chain. We should count (2) as pathological too. Here we are applying a principle which we'll call *Symmetry*: to put it somewhat vaguely for now, we do not treat one expression as pathological and another not without some reason.<sup>15</sup>

Symmetry applies to loops as well as to chains. If Fran produces the expression "the sum of one, two, and the number denoted by Grace's current utterance", and Grace is saying "the sum of one, two and the number denoted by Fran's current utterance", we should ascribe the same semantic status to their utterances. We have no reason to treat one but not the other as pathological.

Later, in Chapter 9, we will provide a rigorous characterization of Symmetry. But we can say a little more here. There are two kinds of symmetric networks: chains and loops. I have given examples of a chain and a loop that involve denotation, but it is easy to construct chains and loops involving extension and truth. In general, chains and loops are infinite sequences of expressions, where each refers to the next. In a chain, no expression repeats; in a loop, every expression repeats. We can represent chains and loops as trees composed of branches and nodes. For example, the expressions (1), (2), ..., (n), ... generate an infinite sequence of sentences, which may be represented as a single-branched tree:

(1)

|

(2)

|

(3)

•

•

•

To determine a denotation for (1), we must determine first the denotation of an expression to which (1) makes reference, namely (2); and to determine a denotation for (2), we must determine first the denotation of an expression to which (2) makes reference, namely (3). And so on, down the infinite branch. No expression repeats on this branch, and this indicates that the expressions form a chain. This is one kind of symmetrical network.

The utterances produced by Fran and Grace may also be represented by an infinite, single-branched tree:

F

|

G

|

F

|

G

•

•

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Both expressions repeat on this infinite branch, indicating that F and G form a loop. This is the other kind of symmetrical network. To anticipate, we will identify F as a singularity of the occurrence of 'denotes' in G, and G as a singularity of the occurrence of 'denotes' in F.

As we will see later, here are far more complicated semantic networks, requiring multi-branched (and even infinitely-branched) trees. But however complicated these networks get, the intuitive idea of a symmetrical network remains the same. A symmetrical network is an infinite sequence of expressions, where each expression makes reference to the next. And either no members of the sequence repeat, or they all do. Once we have rigorously characterized the notion of a symmetric network, we will express the principle of Symmetry this way: *all expressions of a symmetric network are to be treated alike*. Now, the infinite length of the sequence indicates pathology. By Symmetry, then, we treat all members of a symmetric network as pathological.

Consider now an *asymmetric* network. Consider again Fran's and Grace's utterances, and suppose Hugo says:

(H) the number denoted by Fran's utterance.

Then H stands above the loop in which F and G are caught. Consider the tree for H:

H  
|  
F  
|  
G  
|  
F  
|  
G  
.  
.  
.

H does not repeat on this infinite branch: H, F and G do not form a symmetric network. It is no violation of Symmetry to treat H differently from F and G: Symmetry does not constrain Minimality here. Applying Minimality unconstrained by Symmetry, and given the semantic network that G generates, we will *not* identify F as a singularity of 'denotes' in H. The context of H is treated as (non-explicitly) reflective with respect to F. So an evaluation of H will be determined via a reflective evaluation of F, an evaluation that takes into account the pathologicity of F and G. This case will be treated more formally in the next chapter, but it may already be intuitively clear that F's reflectively established denotation is the number 3 (the sum of 1 and 2) So F is not a singularity of the occurrence of 'denotes' in H, and H's denotation is 3.

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On my proposal, the semantic paradoxes are to be treated by the identification and exclusion of singularities. We treat everyday English not as a hierarchy of languages, but as a single language. We do not stratify our semantic predicates; rather we identify singularities of a single, context-sensitive semantic predicate.

Gödel noted that Russell's theory brings in a new idea for the solution of the paradoxes:

It consists in blaming the paradoxes ... on the assumption that every concept gives a meaningful proposition, if asserted for any arbitrary object or objects as arguments.<sup>16</sup>

Gödel goes on to say that the simple theory of types carries through this idea on the basis of a further restrictive principle, by which objects are grouped into mutually exclusive ranges of significance, or types, arranged in a hierarchy.

As we saw in Chapter 1, Gödel suggests that we reject this principle, while retaining the idea that not every concept gives a meaningful proposition for any object as argument:

“It is not impossible that the idea of limited ranges of significance could be carried out without the above restrictive principle. It might even turn out that it is possible to assume every concept to be significant everywhere except for certain 'singular points' or 'limiting points', so that the paradoxes would appear as something analogous to dividing by zero.”<sup>17</sup>

I take my singularity proposal to be very much in the spirit of Gödel's remarks. And we can claim for it the same satisfying feature: our logical intuitions about our semantic concepts are almost correct. It is only in pathological or paradoxical contexts that we may mistakenly suppose that certain expressions denote when they do not, or have a determinate extension when they do not, or have a truth value when they do not -- and in such cases our applications of 'denotes' or 'extension' or 'true' require only *minimal* corrections. A second intuition that requires revision is that our semantic predicates are predicate *constants*. *Repetition,*

*rehabilitation* and *iteration* indicate that with certain changes of context, our semantic terms shift their extension. But these shifts are kept to a minimum.

In correcting both these intuitions, we avoid the problems for the hierarchical approach that I mentioned at the beginning of this chapter. There is no artificial stratification of natural languages; there is no problem about identifying the level of an utterance, because there are no levels; and there are no massive restrictions on the scope of our uses of semantic predicates, just the most minimal restrictions possible.

## Endnotes

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1. When we say, for example, that  $C^*$  is in the extension of 'denotes<sub>cE</sub>', this is a shorthand way of saying that  $C^*$  is the first member of an ordered pair in the extension of any occurrence of 'denotes' that is represented by 'denotes<sub>cE</sub>'. When we talk about the extension of, say, 'denotes<sub>cE</sub>', we are talking about the extension of any occurrence of 'denotes' that is represented by 'denotes<sub>cE</sub>'.
  2. Tarski would not endorse this 'Tarskian' response to paradox in the setting of natural language. As we saw in Chapter 1, Tarski turned away from natural language, and investigated only formal, regimented languages.
  3. For example, Jean van Heijenoort writes: "Today [Richard's] paradox is generally considered solved by the distinction of language levels" (van Heijenoort 1967, p.142.)
  4. An analogous complaint has been lodged against a Tarskian treatment of 'true'; see, for example, Kripke 1975, in Martin 1984, p.57ff. I discuss Burge's and Glanzberg's Tarskian treatment of truth at greater length in Chapter 9.
  5. Russell 1903, p.528.
  6. See Gupta 1982, in Martin 1984, pp.204-205 for parallel worries about the hierarchical treatment of truth.
  7. Kurt Gödel 1944, in Schilpp 1944, p. 149.
  8. Notice that it would be a poor interpretation that unnecessarily implicated your utterance in semantic pathology. In general, speakers do not usually aim to produce pathological utterances, or utterances implicated in paradox. (Of course, philosophical discussions of definability paradoxes provide exceptions to this general rule.)
  9. When we identify  $C$  as a singularity of 'denotes<sub>cC</sub>', we do not introduce denotation gaps (analogous to truth value gaps), or in any way compromise classical logic or semantics. It is straightforwardly false that  $C$  denotes<sub>cC</sub>.
  10. Compare Kripke's claim that the 'level' of an ordinary statement involving truth depends on the empirical facts about the statement, and should not be assigned in advance by the speaker: "in some sense a statement should be allowed to seek its own level" (Kripke 1975, in Martin 1984, p.60. See pp.60-1 and pp.71-2.). Setting aside the notion of level (since the singularity approach does not stratify the denotation predicate), the case of  $N$  is broadly in line with Kripke's claim. The extension of 'denotes' in  $N$  depends on features of the context (the empirical facts about  $N$  spelled out by the semantic network  $N$  generates in its context of use) that need not be known in advance by the speaker.

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11. So if we project from the way we reason in *repetition* and *rehabilitation* discourses, we will have a general recipe for figuring out the truth value of your utterance N. That is sufficient for me to understand your intended meaning, even if you and I don't know the semantic status (paradoxical or otherwise) of the expressions written on the board. Kevin Scharp has suggested that contextual views of truth (and presumably of denotation) fail because "it is often the case that neither the speaker nor the audience knows the levels or paradoxicality status of the targets of the truth attribution" (Scharp 2013, 3.2.4, p.71) – and, according to Scharp, if the contextual account is to succeed, information about paradoxicality must be part of the common ground. In my view, this requires far too much of the speaker and the addressee, and the singularity account does not require it. I can understand what you say when you say N, even if the empirical facts about what is written where and when are not immediately available.

12. In Chapter 6 we'll see another kind of example of a context that is explicitly reflective but not non-explicitly reflective.

13. Some pathological denoting phrases will never get into the extension of 'denotes', even after we have reflected on them. Suppose I write: 'The number denoted by the first eleven words on the board is irrational'. If the phrase in question ('the number denoted by the first eleven words on the board') is the very one I have written, then it is pathological. But, unlike the case of C, the recognition that this phrase is pathological does not enable us to determine a referent for it.

14. Let E be the token occurrence of 'empty extension of an eight-word predicate in U'. Let  $c_U$  be the context of your utterance. The relevant instance of the associated  $c_U$ -schema is this:

$\text{ext}_{c_U}(E)$  is in  $\text{ext}_{c_U}(E)$  iff  $\text{ext}_{c_U}(E)$  is an empty extension of an eight-word predicate in U.

Suppose the left-hand-side is true. Then  $\text{ext}_{c_U}(E)$  is non-empty; so the right-hand-side is false, and so the left-hand-side is false - and we have a contradiction. Suppose on the other hand that the left-hand-side is false. Since E is the only eight-word predicate appearing in U, it follows that  $\text{ext}_{c_U}(E)$  is empty. And then the right-hand-side is true, and so the left-hand-side is true - and we have a contradiction again. E cannot be assessed by the  $c_U$ -schema – E is a singularity of 'extension<sub>U</sub>', and in particular, a singularity of your innocent use of 'extension'.

15. Compare Symmetry with Burge's Principle of Justice (Burge 1979, in Martin 1984, p.110).

16. Kurt Gödel 1944, in Schilpp 1944, p. 149.

17. *Op. cit.*, p. 150.